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An Empirical Analysis of the Disproportionality Theory of Crisis: A Sraffian Approach to the Economic Crisis

Gunwoo Lee

Gunwoo Lee (email: hiphop99dan@korea.ac.kr), Chungnam National University, Republic of Korea

Abstract

Sraffian economics posits that the production prices are uniquely determined within a specific production framework, serving as the benchmark for maintaining economic reproduction. This study aims to link deviations between Sraffian production prices and market prices to economic disproportionality and investigate these divergences to understand the business cycle. Using input-output matrices of the Korean economy, the study computes these deviations using the Steedman and Tomkins (1998) formula. It then assesses the impact of these price changes on economic activity. VAR methodology analysis shows that increasing deviation, indicating disproportionality, negatively affects effective demand in Korea. These findings support Hilferding's (1981) argument that deviations of market prices from labour value (or production price) signal an impending economic crisis. Finally, the research highlights the role of fixed capital, drawing on the crisis theory of Maksakovsky and Tugan-Baranovsky.

Keywords: Sraffa, price deviation, disproportionality theory of crisis

JEL Classification: C32, C51, D57, E22, E32

1. Introduction

Certain economists, notably within the Marxian tradition, have argued that recurring incidence of economic crises within the capitalist economy can be ascribed to inherent vulnerabilities intrinsic to the system. Furthermore, they have argued that the cyclical nature of this phenomenon provides empirical evidence for the existence of internal contradictions, based on analyses such as the long-term declining trend of the profit rate (Fernandez-Aguilera *et al.*, 2022).

It is noteworthy that Tugan-Baranovsky was the earliest proponent to incorporate the reproduction schema proposed by Karl Marx in Volume II of *Capital* as a theoretical instrument for explaining the cyclicality of crises and their underlying origins. Succinctly, his analysis can be encapsulated as follows: "*Tugan*[-*Baranovsky*] *concluded that sustained accumulation depended only on maintaining the appropriate proportional relations between the various branches of production, the implication being that the only possible cause of crises was disproportionality between the branches of production*" (Clarke, 2016, p. 35). Tugan-Baranovsky emphasized sectoral imbalances as a key factor in explaining the disruptive nature of crises within the reproduction of capital. His theory, widely recognized, is often referred to as the disproportionality theory of crisis.

Tugan-Baranovsky's work fundamentally corroborates the existence of an equilibrium condition "*in which exchanges between departments at once satisfied both the material requirements of production and the need for the realization of profits*" (Besomi, 2006, p. 158). Following an exhaustive examination of diverse reproduction scenarios grounded in varying assumptions, Tugan-Baranovsky (1913, as referenced in Besomi, 2006) ultimately articulated his conclusion in the following manner:

"*Provided that production can be expanded and that productive forces are available in a sufficient quantity, demand will expand in the same proportion if social production is proportionately distributed; for, if this condition is satisfied, each new product provides a new purchasing power permitting someone to buy other products*." (p. 158)

Highlighting the ostensibly harmonious portrayal of the capitalist system, "*Tugan*[-*Baranovsky*] *did not believe that proportionality would necessarily be achieved, for the anarchy of the market meant that there were no guarantees that new investment would be appropriately distributed among the various branches of production.*" (Clarke, 2016, p. 35). Indeed, Tugan-Baranovsky (2019, p. 72) advocated the notion that the concept of well-balanced proportions within his illustrative economic framework should be regarded as an ideal construct for comprehending the complexity of capital circulation and the instability of the system in real-world conditions. Accordingly, from his standpoint, it is the resulting disproportionality stemming from the inherent production anarchy within the system that should be considered the primary catalyst for overproduction or crisis.

Although Tugan-Baranovsky provided a persuasive explanation of these phenomena, relative prices do not play a central role in his work as intermediaries or regulators of exchanges among sectors, despite their crucial importance for the sustainability of a capitalist economy. Nonetheless, it is untenable to overlook the role of prices when considering the significance of proportionality across production sectors. As is commonly recognized, Ricardo argued that changes in relative prices alone could induce the reallocation of capital across sectors, thereby restoring proportionality and ensuring the reproducibility of an economy (Clarke, 2016, p. 140). Acknowledging the significance of prices, one might inquire whether distortions in relative prices among industries can provoke a crisis of disproportionality. Importantly, Hilferding (1981) posited that the disjunction between proportions of sectors ought to be assessed within the context of the price structure, which fundamentally governs the production mechanism:

"*The disruption of these proportional relations must be explained in terms of a disruption in the specific regulatory mechanism of production, or in other words, in terms of a distortion of the price structure which prevents prices from giving a proper indication of the needs of production*." (p. 257)

With this objective in mind, the primary aim of this study is to evaluate the explanatory power of the disproportionality theory of crisis through an investigation of price disparities observed within the Korean input-output matrix. The paper is structured as follows: The next section introduces Sraffian prices of production as a benchmark to ensure the reproducibility of the commodity production system. While explicating the role of Sraffian prices of production in the context of reproducibility, we engage in a comparative analysis *vis-à-vis* market prices.

We then present a methodology for computing the deviation between market prices and Sraffian prices of production, employing the input-output coefficient matrix. This methodology, initially developed by Ochoa (1984), is introduced along with a discussion of its inherent limitations in terms of underlying assumptions. An alternative approach, based on the *d*-statistic proposed by Steedman and Tomkins (1998), is subsequently introduced. Thereafter, we argue that the numeraire-free index, pertaining to deviations from the input-output table, may be construed as an appropriate indicator of the extent of distortions affecting reproducibility.

In the empirical analysis segment of this study, we numerically compute the *d*-statistic using data from the Korean input-output (IO) table spanning from 1960 to 2019. The data also form the basis of our structural vector autoregressive (SVAR) analysis, conducted to assess the validity of our hypothesis. Our findings are thoroughly discussed, particularly concerning the empirical support for the disproportionality theory of crisis. In conclusion, we briefly summarize our discussion and empirical findings, outlining directions for future research.

2. Sraffian Prices of Production as a Reference for Reproducibility

In his work, Sraffa (1960) proved that "[t]here is a unique set of exchange-values which if adopted by the market restores the original distribution of the products and makes it possible for the process to be repeated" when the method of production is given (p. 3).[1] Moreover, "*in a system capable of producing a surplus, relative prices are determined on the basis of the conditions of production and the manner in which the surplus is distributed between wages and profits*" (Roncaglia, 1978, p. 4). In the context of a long-period equilibrium, specified by the uniformity of profit rates across all sectors, the resultant "relative prices" are referred to as the Sraffian prices of production.

Sraffa illustrated this concept with an example, as presented in Table 1 below, where the prices of production (*i.e.*, exchange values) are determined, such as 10 quintals of wheat equalling 1 tonne of iron, or 2 pigs. Note that the following tables are constructed based on not monetary terms, but the physical units.

Table 1: Example of three sectors

	Wheat sector	Iron sector	Pig sector	Total
Wheat input	240 q wheat	90 q wheat	120 q wheat	450 q wheat
Iron input	12 t iron	6 t iron	3 t iron	21 t iron
Pig input	18 pigs	12 pigs	30 pigs	60 pigs
Output	450 q wheat	21 t iron	60 pigs	–

Source: Author's own elaboration provided by Sraffa (1960, p. 4)

It is noteworthy that Sraffa's analysis does not necessitate the actual adoption of the relative prices proposed for the three commodities (wheat, iron and pigs) by the market. Rather, the example serves to underline the potential disruption in this economy's proportional relationships whenever the market imposes distinct exchange ratios deviating from the Sraffian prices, the benchmark for reproducibility. Notably, Sraffa's work does not explicitly address the interplay between prices of production and market prices, as has been noted by Roncaglia (1978): "*The fact that Sraffa never talks of an 'economic equilibrium' or of 'equilibrium prices' in relation to his system should also be emphasized. In the absence of any considerations whatsoever of the factors that determine the quantity supplied or the quantity demanded of the various commodities, there is no reason to suppose that prices of production should equate the quantity demanded with*

1 According to Roncaglia (1978), Sraffa's analysis of prices of production emphasizes the concept of reproducibility as a fundamental characteristic of capitalist commodity production. This concept serves as a crucial link between Sraffa's approach and that of classical economists and Marx, while distinguishing it from the marginalist focus on scarcity. Reproducibility refers to the ability of an economic system to sustain and replicate itself over time, given its production conditions and distribution of surplus.

the quantity supplied for any commodity in the long period or that market prices should fulfil this function in the short (or very short) period. In addition, in the absence of any explicit analysis of effective (market) prices the relation between market prices and prices of production must remain undetermined." (pp. 16–17)

In this context, we take market prices merely as the prevailing rates within the market, and they "*are thus representative of deviations from prices of production due to the influence of supply and demand as well as other contiguous causes*" (Roncaglia, 1978, p. 126). In other words, we do not presuppose that market prices inherently tend to converge towards the designated prices of production in advance. Hence, our hypothesis can be formulated as follows: In instances where market prices diverge significantly from prices of production, irrespective of the underlying causes, this disparity between the two price vectors engenders disproportionality among industrial sectors. Such disproportionality has the potential to impede the efficient optimization of overall production capacity. As Marx and Engels (1986) pointed out in *Grundrisse*, workers and capital capacity can become unemployed and idle "*because they are not present in the proportions required by production on the basis of the newly developed productive forces*" (pp. 372–373).

2.1 Disproportionality theory of crisis in the context of Sraffian economics

While Sraffian economics provides a rigorous framework for analysing relative prices, distribution and technological change in capitalist economies, it has been criticized for lacking a comprehensive theory of economic crises (Fine and Harris, 1979; Mattick, 2020). Hahnel (2017) argued that Sraffian theory surpasses Marxian crisis theories by avoiding problematic concepts such as the tendency of the rate of profit to fall. However, it does not offer a fully developed Sraffian alternative theory of crises.

This paper aims to address this gap by developing a disproportionality theory of crisis grounded in Sraffian price theory. By examining deviations between Sraffian prices of production and actual market prices, we can identify growing imbalances and disproportions in the economy that may lead to crises. The Sraffian framework allows us to rigorously measure these deviations and analyse how they relate to economic instability.

Specifically, we argue that significant and persistent divergences between production prices and market prices across sectors indicate a breakdown in the economy's ability to reproduce itself sustainably. These price distortions reflect underlying disproportions in production and demand that can culminate in generalized crises of overproduction. By linking Sraffian microeconomics to theories of macroeconomic instability, we aim to strengthen Sraffian economics as a comprehensive approach to understanding capitalist dynamics and crises.

2.2 Calculating the deviation between Sraffian prices and market prices

Numerous works have attempted to empirically measure the deviations among labour values, market prices and prices of production using data at the national level (Cheng and Li, 2020; Cockshott *et al*., 1995; Işıkara and Mokre, 2022; Ochoa, 1989; Petrovic, 1987; Sánchez and Montibeller, 2015; Shaikh, 1998; Tsoulfidis, 2008; Tsoulfidis and Mariolis, 2007; Tsoulfidis and Paitaridis, 2009, 2017; Tsoulfidis and Rieu, 2006). Research on this subject is mainly based on the methodology developed by Ochoa (1984) and there is little critical debate on this matter except a few works (Díaz and Osuna, 2007, 2009). In the following section, we explore various methods for quantifying deviations between different prices, with a particular emphasis on the detailed examination of chapter four in Ochoa (1984). Next, we criticize the methodology regarding its arbitrariness in the deployed normalization condition.

Ochoa's (1984) methodology

To obtain an exact set of prices of production from the given IO table, the technological relationship between inputs and outputs must be measured in physical units, as in Table 1. However, the IO tables are expressed in monetary terms. In the sense that the prices of production must reflect only the blueprints of a given technique (*i.e.*, method of production), deriving the production prices directly from the matrix evaluated at market prices is problematic and could be criticized as a questionable computation.[2]

In chapter four, Ochoa (1984) introduced the method for measuring the deviation between the Sraffian prices of production and the market prices using the IO matrix evaluated at exogenously given market prices (m). He emphasized that "*we do not have to know … the implicit physical unit we are using. All we require is that this unit remain constant throughout our calculations*" (*ibid*., p. 61).[3] This implies that the market prices utilized remain constant for the duration of the year covered by the IO table. In our research, we regard this assumption as a well-established premise.

2 Despite the lack of substantiation, Han and Schefold (2006) adopted the convention of treating monetary coefficients as if they were in physical units. While this assumption aligns with common practice in many empirical studies and may seem innocuous, we intend to address this issue with greater methodological rigour in the following sections.

3 On the basis of this assumption, Tsoulfidis and Paitaridis (2017) argued against the claim proposed by Díaz and Osuna (2007). Tsoulfidis and Paitaridis (2017) also mentioned Leontief (1986) to justify the assumptions they relied on.

Then, denote the row vector of the Sraffian prices of production, real wage bundle, gross output and unit cost of labour by s, b, q and a_0, respectively.[4] Further, A is the physical input-output coefficient matrix, where its elements a_{ij} denote the quantity of the commodity i needed to produce a single unit of the commodity j, and μ is the rate of profit. Then, the Sraffian price equation can be derived as follows:

$$s = sb'a_0 + \left(1+\mu\right)sA \tag{1}$$

Let us post-multiply (1) by $[m^{-1}]$.[5] Then, we can obtain the following equations:

$$s\left[m^{-1}\right] = s\left[m^{-1}\right]\left[m\right]b'a_0\left[m^{-1}\right] + (1+\mu)s\left[m^{-1}\right]\left[m\right]A\left[m^{-1}\right] \tag{2}$$

With the definitions of $\hat{\boldsymbol{s}} = s[m^{-1}]$, $\hat{b} = [m]b'$ and $\hat{A} = [m]A[m^{-1}]$[6] we have the following price equation concerning $\hat{\boldsymbol{s}}$, which is the Sraffian prices of production per market currency unit for each sectoral output:

$$\hat{\boldsymbol{s}} = \hat{\boldsymbol{s}}\hat{b}'\hat{a}_0 + (1+\mu)\,\hat{\boldsymbol{s}}\hat{\mathbf{A}} \tag{3}$$

Note that $\hat{\mathbf{A}}$ is the input-output coefficient matrix evaluated at the relative market price (m_i/m_j), which is normally available, as we explained earlier. The results are obtained by calculating the ratio of the input value evaluated at m_i to the total output value evaluated at m_j for every sector. If we rearrange Equation (3) for the expression of $\hat{\boldsymbol{s}}$, we obtain the following eigen-equation for matrix $\hat{\mathbf{A}}$:

$$\frac{1}{\left(1+\mu\right)}\hat{\boldsymbol{s}} = \hat{\boldsymbol{s}}\hat{\mathbf{A}}\left(I-\hat{b}'\hat{a}_0\right)^{-1} \tag{4}$$

Equation (4) is the problem of eigenvalue for matrix $\hat{\mathbf{A}}\ (I - \hat{b}\ \hat{a}_0)^{-1}$ and $\hat{\boldsymbol{s}}$ is defined up to a scalar. Then, we need a normalization condition to have a unique set of prices of production, $\hat{\boldsymbol{s}}$. Ochoa suggested the condition $\Sigma_j\, \hat{s}_j\, \hat{q}_j = \Sigma_j\, \hat{q}_j$ ($\hat{q}_j$ is the monetary amount of output for the sector j), which is consequently equal to the equation $\Sigma_j\, (s_j/m_j)m_jq_j = \Sigma_j\, m_jq_j$. Then, the following equation holds:

$$\sum_j s_j\, q_j = \sum_j m_j\, q_j \tag{5}$$

Ochoa (1984) did not explicitly elucidate the rationale behind the normalization condition outlined in Equation (5); however, he clarified its intended objective, which is "*to set the price*

4 Then, sb' is a scalar of real wage (ω) for workers.

5 [ ] indicates a diagonalization of the given vector.

6 The matrix $\hat{\mathbf{A}}$ is made up of elements $\hat{a}_{ij} = a_{ij} \times m_i/m_j$ and the vector $\hat{\boldsymbol{s}}$ consists of $\hat{s}_j = s_j/m_j$.

of output in terms of computed prices equal to the price of output in terms of market prices" (p. 55). Moreover, the normalization condition aids in comparing the Sraffian production prices to market prices by standardizing market prices to unity ($m_j = 1$) for each industry j.

To investigate the normalization condition above, let us suppose that the true market prices are given as m = ($1.1; $9; $5) in the economy described in Table 1. With presumed knowledge of the market prices associated with each commodity, we possess the capability to formulate the monetary representation of the IO table, as demonstrated in Table 2.

Table 2: Example IO table evaluated at a given market price vector

	Wheat sector	Iron sector	Pig sector
Wheat input	264	99	132
Iron input	108	54	27
Pig input	90	60	150
Output	$495	$189	$300

Source: Author's own elaboration

Then, to derive the IO coefficient matrix ($\hat{\mathbf{A}}$), we must divide the value of the inputs by the value of the outputs for each sector. In the following matrix $\hat{\mathbf{A}}$, for example, the number 0.5238 in row 1, column 2 refers to the size of wheat valued in dollars that must be put into producing a dollar's worth of iron. Note that $\hat{a}_{12} = a_{12} \times \left(\frac{m_1}{m_2} \right)$.

$$\hat{\mathbf{A}} = \begin{bmatrix} 0.5333 & 0.5238 & 0.44 \\ 0.2182 & 0.2857 & 0.09 \\ 0.1818 & 0.3175 & 0.5 \end{bmatrix}$$

According to Ochoa, following the normalization condition $\sum_j \hat{s}_j \hat{q}_j = \sum_j \hat{q}_j$, the ratio between the Sraffian prices of production and market prices can be obtained, in which the latter is assumed to be unity. The numerical result from the available input-output coefficient matrix $\hat{\mathbf{A}}$ is $\hat{s}$ = (0.9318; 1.1389; 1.0250). The wheat sector exhibits a lower production price relative to its corresponding market price, whereas the reverse holds true for the iron and pig sectors. Furthermore, Ochoa introduced indices aimed at summarizing the comprehensive divergence between the two price vectors, namely, the mean absolute deviation (MAD) and the mean absolute weighted deviation (MAWD). By applying these metrics, it becomes evident that the MAD stands at 7.74% and the MAWD registers at 6.86% in this instance, as computed through the following calculations:

$$\text{MAD} = \frac{1}{n} \times \sum_{i=1}^{n} \left| m_i - s_i \right| \Big/ m_i$$

$$\text{MAWD} = \frac{m_i q_i}{\sum_j m_j q_j} \times \sum_{i=1}^{n} \left| m_i - s_i \right|$$

Criticism of Ochoa's (1984) methodology

Let us suppose that a researcher who wants to measure the deviation between the Sraffian prices of production and market prices chooses the normalization condition that is $\Sigma_j s_j \psi_j$ with $\psi = (1\ 1\ 1)$. In other words, we set equality between the sum of the Sraffian prices of production and the sum of the market prices. By performing the same method, the following results can be obtained: s' = (0.94375; 9.4375; 4.71875) and $\hat{s}$ = (0.8580; 1.0486; 0.9438). Here, the MAD is 8.23% and the MAWD is 9.79%. In contrast to the preceding scenario, the production price in the pig industry is lower than the corresponding market price. This outcome stems from shifts in the absolute values of production prices, the occurrence of which can be attributed to the different normalization condition. The thought experiment conducted here highlights several critical considerations that should be emphasized in the computation of deviations.

Firstly, it is crucial to underline that the true market prices used to construct the available IO tables remain unknown. While we accept the premise that these elusive market prices remain constant during the analysis period, it cannot be justified that in a specific sector the ratio shifts from exceeding unity to falling below it, or vice versa, solely due to alterations in the normalization condition used to delineate a distinct set of production prices.[7] Secondly, due to the inability to precisely ascertain the ratios for each sector, the indicators are unable to detect situations where market prices consistently outweigh production prices across the entire economic spectrum or vice versa. Thirdly, the exact magnitude of the deviation between production prices and market prices across the entire economy remains ambiguous. Such uncertainty stems from the introduction of arbitrary normalization conditions.

Our discussion is also supported by Steedman and Tomkins (1998, p. 385): "*When measuring the deviation of prices from values … one must ensure that the measure used is theoretically appropriate … and not influenced by extraneous considerations.*" In the next section, we discuss

7 Tsoulfidis and Rieu (2006) contrasted how the overall deviations and those of each sector change when the normalization conditions differ to discuss the empirical evidence concerning the "new interpretation". In 1995 and 2000, for example, the prices of production in the non-metallic mineral industry were reported to be lower than the market prices with normalization condition Equation (5) but *vice versa* with $\Sigma_j s_j y_j = \Sigma_j m_j y_j$, where y indicates the net output.

the measure, called the *d*-statistic, proposed by Steedman and Tomkins and its implication in relation to “reproducibility”.

2.3 *D*-statistic and reproducibility

Calling attention to the questionable normalization condition, Steedman and Tomkins (1998) suggested the “*d*-statistic”, which is computed as $d = \sqrt{2(1 - \cos\theta)}$, where θ means the angle between the two price vectors that we want to compare. The measure is independent of a normalization condition when calculating the deviation between two prices; the *d*-statistic is numeraire-free.

Moreover, we focus on the *d*-statistic because it measures in particular the extent to which “the *directions* of the two vectors differ from one another” (Steedman and Tomkins, 1998, p. 380). The crux of reproducibility lies in the interplay of exchange ratios or the relative price structure among commodities, rather than the absolute magnitudes of individual commodity prices. As reproducibility hinges on the angular relationship between two price vectors, we can indirectly gauge the extent of disruption to reproducibility within a given economy by using the estimated angle. Furthermore, this index remains impervious to the influence of arbitrary normalization conditions, rendering the *d*-statistic a reliable proxy for quantifying the distortions in reproduction between Sraffian prices of production (established as the benchmark for reproducibility) and market prices (subject to diverse contingent factors).

Figure 1 describes the market price and the Sraffian price of production, the latter of which is generally argued to be a benchmark for the market price. In the following section, we assume the discrepancy between the prices of production and the market prices as an indicator of disproportionality in an economy at a specific time. Note that Sraffa’s analysis of prices of production encompasses the role of time as Roncaglia (1978) pointed out:

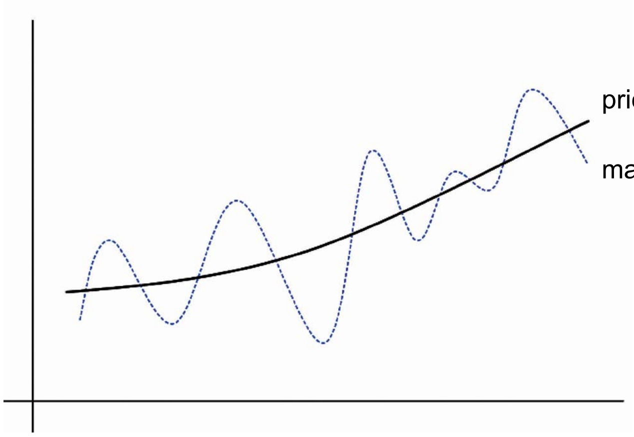
“*It would seem more correct to say that Sraffa’s analysis is not static, but rather that it represents a ‘photograph’ of a particular moment of a system’s development. This is something rather different for it does not abstract from time. Instead, time is taken into account by the fact that any particular moment of time is determined by its past history, and serves as the determining factor of the next moment in time*.” (p. 119)

In this context, the computation of the *d*-statistic over an extended temporal span enables an assessment of the alterations in the economy’s reproducibility over an extended period.[8]

8 Díaz and Osuna (2009) argued that it is not possible to properly calculate the deviation between price vectors, even with the *d*-statistic, if we do not know the exact physical unit used. However, they “*do not claim that the measures of the price-to-price deviation examined in this paper cannot be used in the context of time series data, but only that they cannot be used in the context of cross-sectional data*” (p. 438).

Additionally, it holds significance to scrutinize the dynamic interplay between this index and other macroeconomic variables as a means of evaluating the credibility of the theory of crisis established on disproportionality.

Figure 1: Market price and price of production



Source: Author's own elaboration

3. Disproportionality Theory: Empirical Analysis

3.1 Computing *d*-statistic using Korean input-output matrix: 1960–2019

Following the approach of Tsoulfidis and Rieu (2006), we calculate the *d*-statistic to quantify the disparities between market prices (MP) and Sraffian production prices (SP) in Korea. To determine the *d*-statistic, we employ eigen-equation (6):

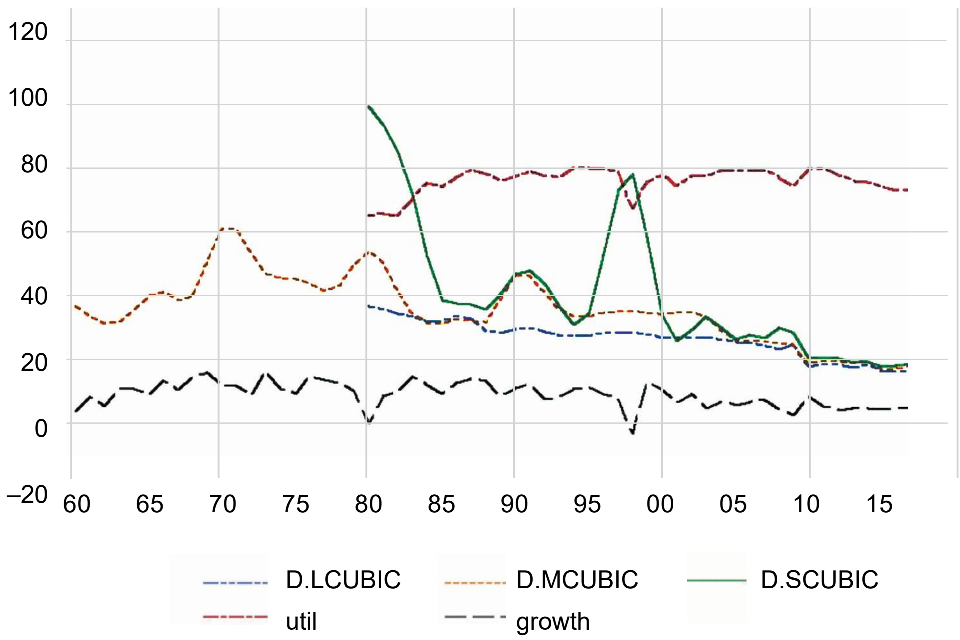
$$\frac{1}{(1+\mu)}\hat{s} = \hat{s}\hat{A}\left(I - \hat{D} - T - \hat{b}'\hat{a}_0\right)^{-1} \tag{6}$$

We consider adding two more pieces of information: the diagonal matrix **T** has the net producer tax coefficients and $\hat{D}$[9] indicates the square matrix of depreciation coefficients of the fixed capital that is evaluated at market prices. The Perron–Frobenius theorem implies that the largest eigenvalue of (6), 1 / (1 + μ), "will be the only one associated with a unique positive left-hand eigenvector defined up to a multiplication by a scalar" (Tsoulfidis and Rieu, 2006, p. 280), The

9 *"The fixed capital depreciation square matrix is constructed by multiplying the column vector of gross fixed capital formation distribution … with the row vector of depreciation coefficients."* (Cheng and Li, 2020, p. 119). An implicit assumption in this method is that all sectors are replaced by capital goods whose distribution is equal to the economy-wide distribution of fixed capital formation.

fixed capital depreciation square matrix is constructed by multiplying the column vector of gross fixed capital formation distribution … with the row vector of depreciation coefficients" (Cheng and Li, 2020, p. 119). An implicit assumption in this method is that all sectors are replaced by capital goods whose distribution is equal to the economy-wide distribution of fixed capital formation. and it yields the corresponding Sraffian prices of production per currency unit for each sectoral output.[10] Subsequently, using an arbitrary normalization condition, we derive a set of uniquely defined production price vectors, to be used in calculating the *d*-statistic.[11]

Figure 2: *D*-statistic (small, middle and large), growth rate and utilization rate



Source: Author's own elaboration

Figure 2 shows the computed three kinds of *d*-statistic from the Korean IO tables from 1960 (or 1980) to 2019.[12] As Tsoulfidis and Rieu (2006) noted, the Korean IO tables categorize industries into three levels: large, medium and small. Before moving on, it is crucial to contem-

10 Note that "*in this case the rate of profit is at its maximum possible level*" (Pasinetti, 1977, pp. 76–77).

11 After we compute $\theta = \tan^{-1}\left[std\left(\hat{s}\right) / mean\left(\hat{s}\right)\right]$, then $d = \sqrt{2\left(1 - \cos\theta\right)}$ can be derived. See more details in Steedman and Tomkins (1998, pp. 380–382). The std and mean are the standard deviation and the arithmetic average of the elements in the vector, respectively.

12 In 2003, the Bank of Korea initiated the annual publication of the input-output table. To monitor year-to-year trends comprehensively, the graph presented above was generated using cubic spline interpolation applied to the computed outcomes derived from data published approximately biennially or triennially. Furthermore, the computation of the *d*-statistic rests on the assumption that all capital invested in each industrial sector is fully utilized annually, presupposing that all branches of production operate on a circulating capital basis.

plate several key considerations. Firstly, in the large and medium classes, consistency decreases as the number of industries increases. Conversely, the smallest class has maintained a stable industry count (160 to 166) since 1980, offering greater reliability for time series analysis.

Secondly, for a comprehensive assessment of economic disparities, using datasets that include more industries is recommended, as they facilitate a deeper examination of interdependencies and a more accurate measurement of economic disproportionality. The *d*-statistic based on middle-level disaggregated data offers distinct advantages, covering the longest time span from 1960 to 2019 and showing periodic patterns in disproportionality with heightened variability. In contrast, the *d*-statistic from the large-scale classification appears remarkably stable. This suggests that a higher level of industry granularity enhances the informativeness of the *d*-statistic.

Despite the extended temporal coverage offered by the *d*-statistic derived from the middle-scale classification, our preference lies in the utilization of the index calculated from the IO table of the small-scale classification, which provides the most intricate and detailed insights. As shown in Figure 2, the solid line representing the *d*-statistic computed using the small-scale classification (D_SCUBIC) exhibits more pronounced fluctuations and appears to exhibit a negative correlation with variables such as GDP growth rate and utilization rate.[13]

Figure 2 illustrates the sensitivity of the *d*-statistic to major economic crises, highlighting its significance as a measure of disproportionality. For example, the *d*-statistic peaks during the 1998 IMF crisis, indicating severe imbalances, and shows notable increases during the 2003 Korean credit card crisis. While the *d*-statistic rises in response to the 2008 global financial crisis, its relative increase is smaller than in previous crises, likely due to the external origin of the shock in the United States. Notably, given that our study's IO table is based on Korean data, external crises such as the 2008 financial crisis may not be entirely captured. These patterns, nevertheless, suggest a potential relationship between *d*-statistic fluctuations, indicating underlying disproportionality, and broader economic disruptions. By connecting these findings to contemporary economic crises, this study underscores the relevance of disproportionality theory, suggesting that monitoring *d*-statistic variations over time could offer valuable insights into an economy's stability and resilience under various pressures.

13 Figure 2 illustrates a discernible trend wherein all three categories of the *d*-statistic exhibit a propensity for gradual decrease over a temporal span. Further elaboration on this observation is presented briefly in the concluding section.

3.2 SVAR analysis of disproportionality theory of crisis

In this subsection, we employ the structural vector autoregressive (SVAR) approach to examine the interdependencies between the *d*-statistic, representing the level of overall economic disproportionality, and variables including the utilization rate, growth in fixed capital, profit rate and export ratio. Hence, the model is designed to incorporate multiple dependent variables, each of which includes lagged variables as explanatory factors. Nevertheless, as shown in the next section, this model exhibits a constraint in that it necessitates a well-defined causal relationship between the variables, particularly when imposing short-term restrictions to resolve the identification problem. In the next subsection, we explain our approach to identification and substantiate the predetermined ordering among the variables, drawing upon various scholarly sources for justification.

SVAR

First, the reduced form of VAR(p) model is defined as follows (for simplicity, we exclude a constant term):

$$\boldsymbol{y}_t = \mathbf{A}_1\boldsymbol{y}_{t-1} + \ldots + \mathbf{A}_\mathbf{p}\boldsymbol{y}_{t-p} + u_t \tag{7}$$

In Equation (7), $\boldsymbol{y}_t = (y_{1t}, \ldots, y_{kt})$' is a vector of endogenous variables, $\mathbf{A}_\mathbf{i}$ is a $(k \times k)$ square matrix composed of coefficients and u_t is a k-dimensional white noise process. Note that $E(u_t u_t') \equiv \Sigma$ and that Σ is assumed to be non-singular but it does not require it to be a diagonal matrix.[14] The latter implies that, in its reduced form, it permits contemporaneous correlation among the errors, thereby rendering an unexpected disturbance in an element within as an uncontrolled factor for observing the dynamic relationships among $(y_{1t}, \ldots, y_{kt})$.

Because the innovations in u_t are correlated contemporaneously, it is required to introduce a matrix **B** that is designed to encompass contemporaneous interactions. Let us consider the so-called structural VAR:

$$\mathbf{B}\boldsymbol{y}_t = \mathbf{B}_1\boldsymbol{y}_{t-1} + \ldots + \mathbf{B}_\mathbf{p}\boldsymbol{y}_{t-p} + e_t \tag{8}$$

To express the contemporaneous correlation among endogenous variables, $E(e_t e_t') \equiv \Omega$ must be a diagonal matrix. By introducing this new matrix, we can now say that the simultaneous correlation between the innovations in e_t has been controlled; e_t is not cross correlated. Thus,

14 To estimate Equation (7) consistently, we need the assumption that u_t is not correlated with past values, $(y_{t-1}, \ldots, y_{t-p})$.

Equation (8) considers the contemporaneous relationship among the variables in $\boldsymbol{y}_t$. Suppose **B** is invertible. Then, the general structure of Equation (8) can be rewritten as follows:

$$\boldsymbol{y}_t = \mathbf{B}^{-1}\mathbf{B}_1\boldsymbol{y}_{t-1} + \ldots + \mathbf{B}^{-1}\mathbf{B}_\mathbf{p}\boldsymbol{y}_{t-p} + \mathbf{B}^{-1}e_t \tag{9}$$

Compared to Equation (8), we can easily see that $u_t = \mathbf{B}^{-1}e_t$ and $\mathbf{A_i} = \mathbf{B}^{-1}\mathbf{B_i}$ hold. The main issue here is that numerous structural VAR from Equation (9) can be reduced to a unique Equation (7); although we can consistently estimate a reduced form of VAR from Equation (7), additional restrictions must be imposed to recover the parameters in the structural form from which one can derive the impulse response functions. While we have $pk^2 + \frac{k(k+1)}{2}$ parameters to estimate in the reduced form of Equation (7), there are $k + (p + 1)k^2$ parameters in the structural VAR. Therefore, the number of restrictions required to identify the structural model is $\frac{k(k+1)}{2}$. With the additional assumption, $E(e_t e_t') \equiv \Omega$ being the unit variance, there remain $\frac{k(k-1)}{2}$ restrictions to matrix **B**. Hence, researchers are required to opt for a method of imposing constraints accompanied by sound and rigorous justifications.

For example, the restriction most commonly used is imposing **B** to be a lower triangular matrix. In doing this, we have imposed $\frac{k(k-1)}{2}$ restrictions and the structural model is uniquely identified. If we perform the Cholesky decomposition to the estimated variance–covariance matrix $\hat{\mathbf{\Sigma}}$, we have $\hat{\mathbf{\Sigma}} = LL'$ and let $\mathbf{B}^{-1} = \mathbf{L}$ where **L** is a lower triangular matrix. Then, the structural model can be recovered, because $\mathbf{B}u_t = e_t$ and $\mathbf{BA_i} = \mathbf{B_i}$. The identification method described here is referred to as a short-run or recursive restriction. Furthermore, various methods have been proposed to identify the structural model, such as sign restriction and long-run restriction.[15] Nevertheless, we opt for the recursive restriction due to our analytical focus on ascertaining whether the level of the activity variable, specifically the utilization rate, undergoes a persistent or transitory decline in response to variations in the degree of disproportionality. In the following subsection, we introduce and elucidate our primary variables. Afterwards, we delve into the proposed causal relationships among these endogenous variables to assess the theory of disproportionality-driven crises.

15 The approach proves particularly advantageous when scrutinizing economic policies, such as monetary policy, as exemplified in the work of Blanchard and Quah (1989), as it effectively neutralizes the accumulative impact of a specific variable. In the traditional macroeconomic model, this methodology can be deemed both practical and theoretically justifiable, given established paradigms such as monetary neutrality.

Data selection and rationale for ordering

In our analysis, we have chosen the following variables to grasp the dynamics of disproportionality: the ratio of exports to nominal GNI, profit rate, growth rate of fixed capital, *d*-statistic and utilization rate in manufacturing industries.[16] As previously deliberated, the establishment of a causal chain to determine the order among these five variables is crucial for the identification. Initially, we formulate the orderings as follows: **log (exports) → rate of profit → growth rate of fixed capital → *d*-statistic → log (utilization rate)**. Then, our choice for the restriction is equivalent to the following expression of Equation (10), $u_t = \mathbf{B}^{-1} e_t$:

$$\begin{bmatrix} u_{netex} \\ u_{profit} \\ u_{rfixed} \\ u_{d-stat} \\ u_{util} \end{bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 & 0 \\ b_{31} & b_{32} & b_{33} & 0 & 0 \\ b_{41} & b_{42} & b_{43} & b_{44} & 0 \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} \end{bmatrix} \begin{bmatrix} e_{netex} \\ e_{profit} \\ e_{rfixed} \\ e_{d-stat} \\ e_{util} \end{bmatrix} \tag{10}$$

Exports (*lexp*): We apply a logarithmic transformation to the ratio of exports to GNI (hereinafter referred to as exports), positing its exogenous determination with respect to all other variables. This implies that the magnitude of exports remains unaltered by concurrent influences from the remaining four variables. Particularly noteworthy is the consideration of foreign trade variables in our analysis, a salient point given Korea's status as a relatively small, export-dependent economy. By integrating this variable into our empirical analysis, our objective is to manage and control the potential impact of external global shocks on the Korean economy.

Profit rate (*profit*). We regard the profit rate as a measure reflecting the distribution of income between labour and capital. In the context of the Sraffian price framework, alterations in the profit rate lead to corresponding adjustments in production prices. Although Sraffa (1960) initiated his price analysis with an exogenously specified wage rate, he conceded:

"*The rate of profits, as a ratio, has a significance which is independent of any prices and can well be 'given' before the prices are fixed. It is accordingly susceptible of being determined from outside the system of production, in particular by the level of the money rates of interest.*" (p. 33)

Hence, we treat the profit rate as an exogenous variable with respect to all other variables, except exports. Variations in the profit rate[17] invariably lead to modifications in the configuration

16 All data employed in this analysis originate from Bank of Korea's (BOK) Economic Statistics System.

17 We compute the profit rate as the quotient of operating profit for non-financial corporations divided by their productive capital stock.

of production prices, acting as a foundational element for reproducibility. Consequently, it is natural to postulate that it would systematically affect the *d*-statistic.

Growth rate of fixed capital (*rfixed*). Establishment of fixed capital or equipment investments likely hinges on the prevailing rate of profit, as substantial investments are typically made with present and expected profitability. Furthermore, it is reasonable to postulate that the growth rate of fixed capital would be affected by the fluctuation of the export level. Nevertheless, some readers may question the selection of this specific variable. Including the growth rate of fixed capital in our analysis aims to examine its impact on overall economic proportionality, as highlighted by Hilferding:

"*For Hilferding the new stage of capitalism was defined as being under the domination of finance capital, which expressed the integration of bank and industrial capital under the domination of the banks. This development was determined primarily by the substantial increase in the importance of fixed capital, which tied up large amounts of capital for long periods. This reduced the mobility of capital, and so its flexibility in response to economic fluctuations and emerging disproportionalities, and this barrier to the equalisation of supply and demand was for Hilferding the source of crises.*" (Clarke, 2016, p. 40)

***D*-statistic (*d-stat*)**. We employ the *d*-statistic derived from the most detailed classification of IO tables spanning the years from 1980 to 2019 (refer to Figure 2). This metric serves as a proxy for the comprehensive level of disproportionality and is posited to be contemporaneously influenced by exports, the rate of profit and the growth rate of fixed capital. The following quotation provides insight into the potential impact of increased fixed capital on the extent of economic disproportionality:

"*The inevitability of expansion growing over into crisis results from the character of the capitalist system's 'regulation' when it is under pressure from the massive renovation of fixed capital. This factor, deriving from the uneven progress of technology in a capitalist economy, gives birth to the unique waves of market competition that cause long deviations of market price from the price of production and thus alter the 'proportions' between parts of the developing system. The result of an outbreak of generalised overproduction in the market is a crisis.*" (Maksakovsky, 2002, p. 191)

Utilization rate (*lutil*). The logarithmically transformed utilization rate is presumed to be the most endogenous variable, signifying that it is concurrently influenced by the four aforementioned variables, including itself. This assumption aligns with reasonable economic dynamics.

Economic crises, often characterized by surpluses or excess in the production of goods and services, are often mirrored by a marked decline in the utilization rate within the manufacturing sector. This reduction can be construed as indicative of an increase in unutilized industrial capacities and a surplus in the labour force specific to this sector. As a gauge of economic activity, we have opted to employ the utilization rate as a surrogate figure. Our primary objective revolves around discerning whether an unforeseen increase in the *d*-statistic has an adverse effect on the utilization rate, a scenario that has significant consequences for the broader economy.

Before moving on to our next analysis, it is important to assess the existence of unit roots within each variable, because "*the impulse response functions involving nonstationary variables can have very large standard errors*" (Enders, 2014, p. 336). Table 3 shows us the results of augmented Dickey–Fuller (ADF) and Phillips–Perron tests.

Table 3: Unit root tests

Variable	Augmented Dickey–Fuller		Phillips–Perron	
	***t*-statistic**	***p*-value**	***t*-statistic**	***p*-value**
lexp	–3.1317 (0)**	0.0305 (C)	–3.0864**	0.0340 (C)
nprofit	–1.6601 (0)	0.1023 (N)	–1.7504*	0.0760 (N)
rfixed	–4.6395 (1)***	0.0027 (T)	–2.8435	0.1896 (T)
d-stat	–3.4769 (2)*	0.0568 (T)	–2.8815*	0.0567 (C)
lutil	–3.4337 (0)**	0.0156 (C)	–3.4208**	0.0161 (C)

Note: In the unit root test, T denotes the inclusion of both a significant constant term and trend, C signifies the inclusion of a significant constant term alone, and N indicates the exclusion of both constant and trend terms if they are found to be insignificant. The Phillips–Perron unit root test employs a quadratic spectral kernel with an Andrews bandwidth, while the lag selections within the ADF results are determined by the Schwarz information criterion. ***, ** and * imply that the null hypothesis that the variable has a unit root is rejected at the significance levels of 1%, 5% and 10%, respectively. Each test encompasses the entire sample corresponding to the respective variable.

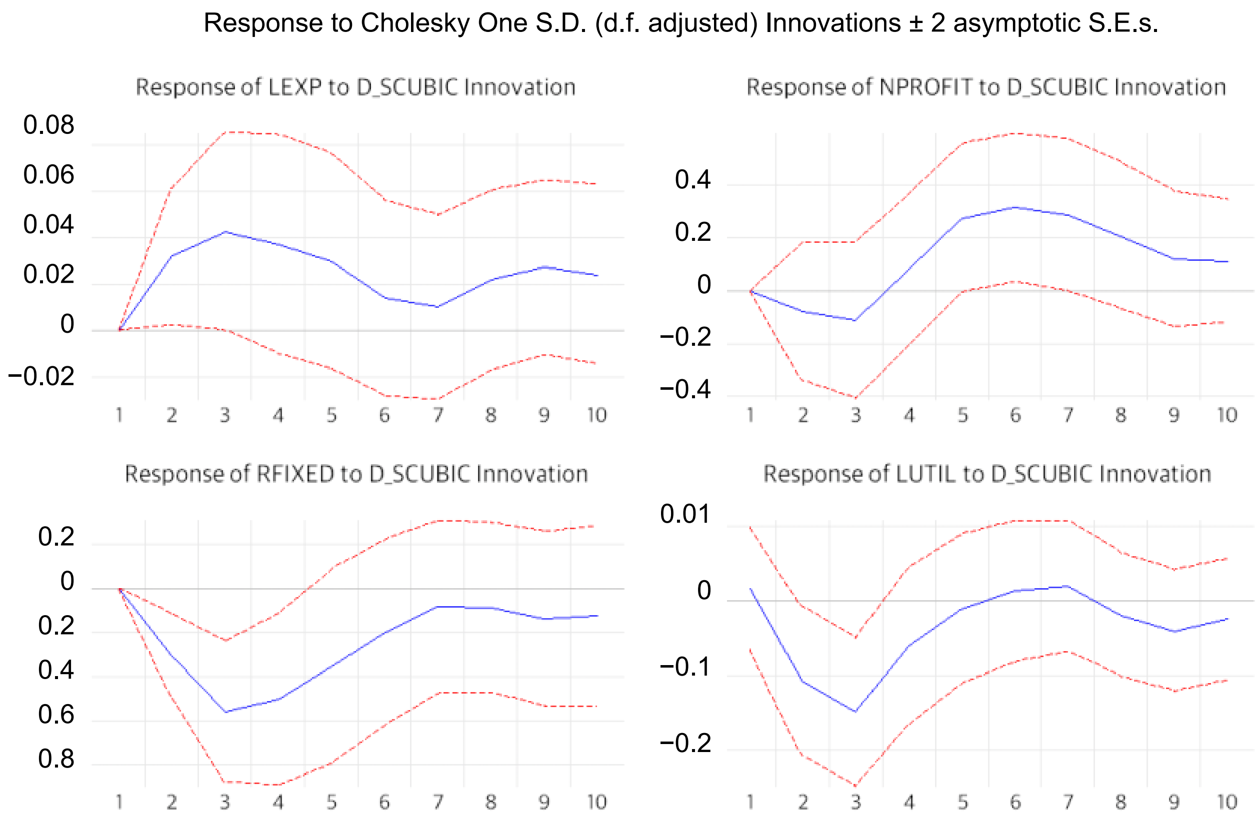
Source: Author's own calculations

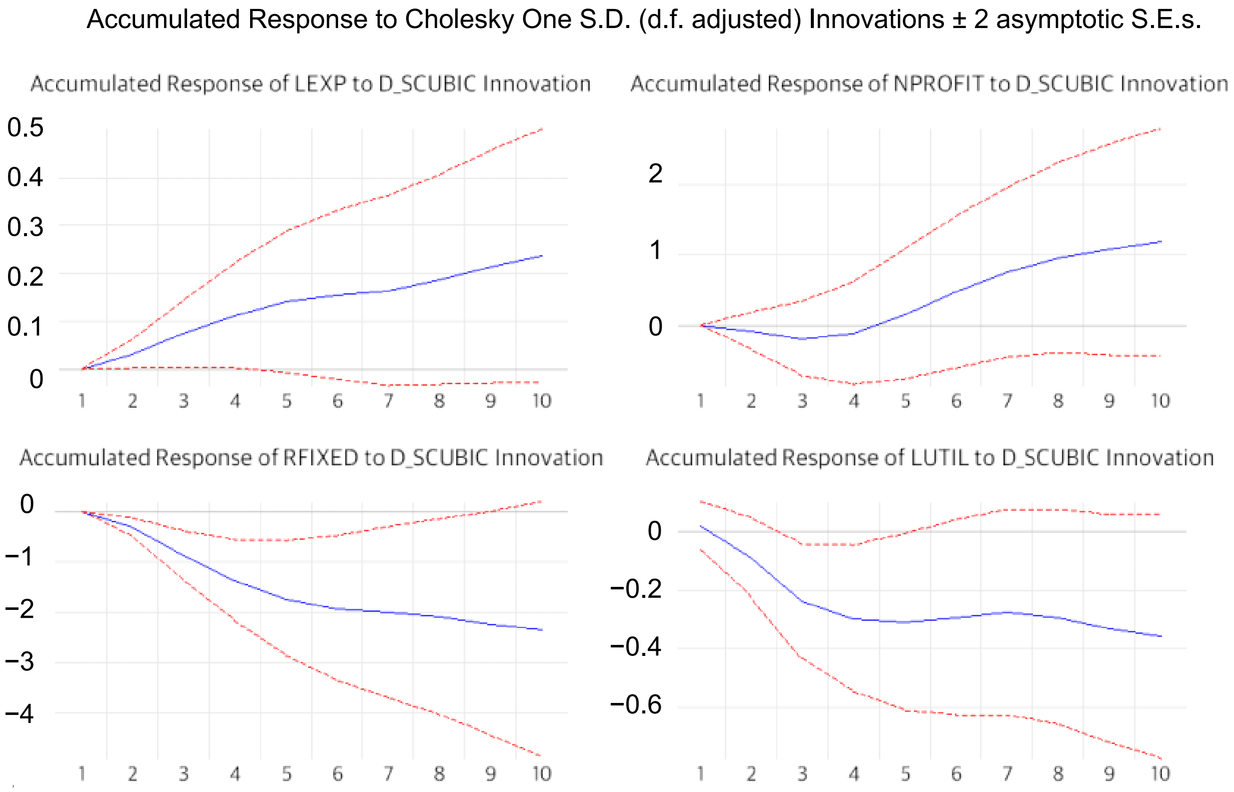
Table 3 indicates that variables do not exhibit unit roots in general, thereby enabling us to move on towards the analysis of impulse response functions. Nonetheless, it is noteworthy to highlight that "*under the short-run identification scheme, the restricted (based on pretests for unit roots and cointegration) and unrestricted VAR specifications do not exhibit substantial differences in their computed impulse responses.* (…) *However, the unrestricted VAR in levels appears to be the most robust specification when there is uncertainty about the magnitude of the largest roots and co-movement between the variables*." (Gospodinov *et al*., 2013)

While instances indicating the presence of unit roots were identified, it is significant to mention that conflicting outcomes were observed across various model specifications, including those involving the inclusion of trends or constant terms. Instead of relying on the uncertainties of unit root testing, we proceeded with our analysis guided by the conclusions of the aforementioned study, which advocate for the robustness of the VAR model when using level variables in short-term restrictions. Furthermore, in the Appendix, we provide the results of stability tests conducted on our VAR model, presenting the inverse roots of AR characteristic polynomials. The result implies that the whole sequences "will be jointly covariance stationary if the stability condition holds. Each sequence has a finite and time-invariant mean and a finite and time-invariant variance" (Enders, 2014, p. 288). We have determined a lag length of 3 for our VAR model, as higher orders did not meet the stability criteria. The Appendix includes the results of lag length criteria tests, which predominantly substantiate our selected lag length.

3.3 Results

Figure 3: (Accumulated) response functions to impulse of *d*-statistic



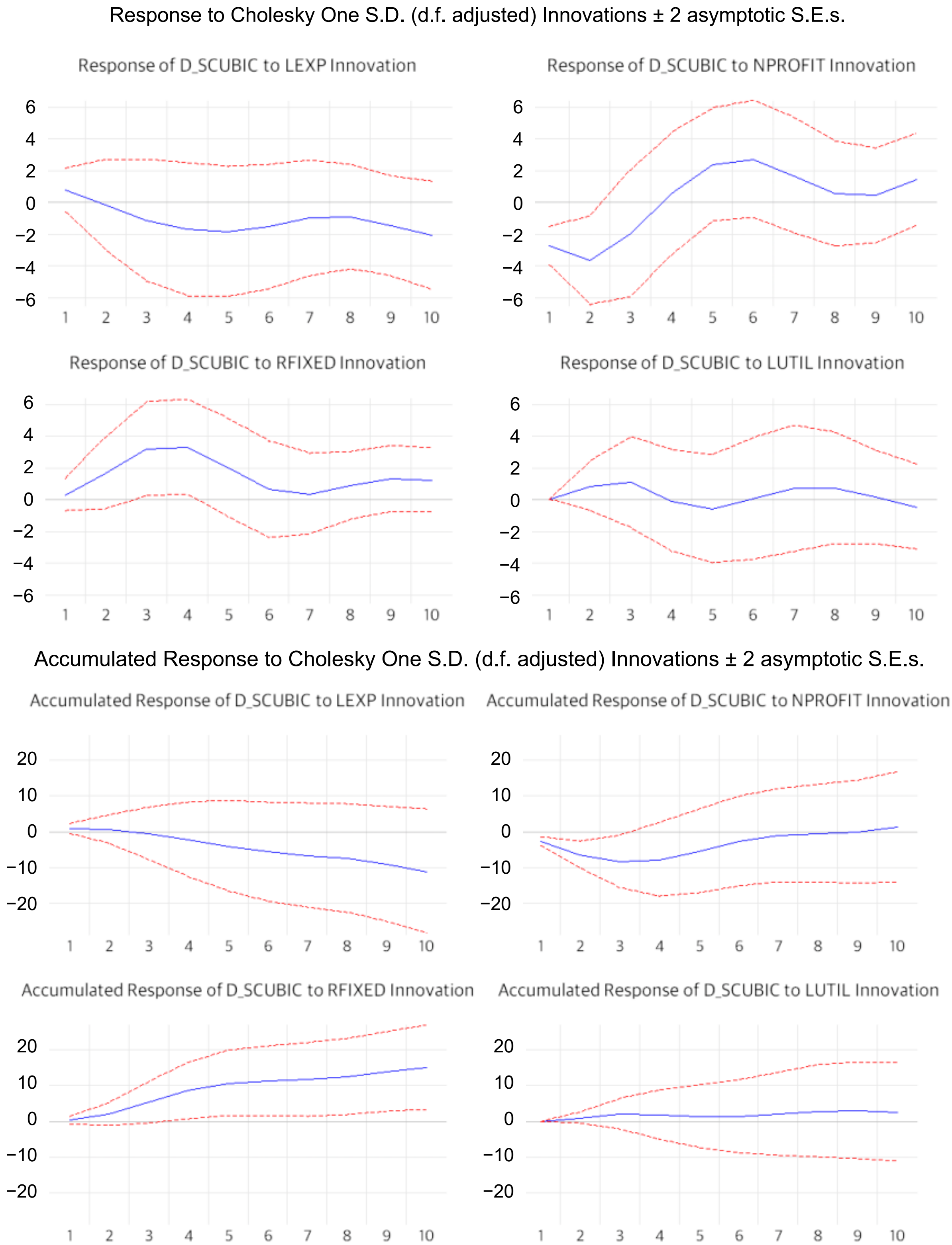


Source: Author's own elaboration

Figure 3 offers a graphical representation illustrating the dynamic relationship between the *d*-statistic and other variables, spanning a duration of 10 periods. The impulse response function unveils a notable and statistically significant adverse impact of an increased *d*-statistic on the utilization rate, particularly evident during the 3rd and 4th periods. This suggests that the deepening disproportionality has a discernible negative effect on the utilization rate through an interplay among various macroeconomic variables.

Furthermore, the accumulative response function delineates that the adverse effects of disproportionality on the utilization rate last for approximately the 5th periods following the initial upsurge in the *d*-statistic. The empirical results are consistent with our theoretical hypothesis, suggesting that disruptions in proportionality lead to adverse consequences characterized by heightened idleness of production facilities. These findings imply that an increase in disproportionality among production sectors within the economy enhances the likelihood of what can be termed "overproduction" in the classical sense. Furthermore, even though the effect becomes statistically insignificant after the 6th period, it is reasonable to anticipate that disturbances in proportionality among production sectors have a prolonged negative impact on economic activity in the long term.

Figure 4-1: (Accumulated) response of *d*-statistic to impulse of other variables



Source: Author's own elaboration

Let us make a more thorough examination of the interplay between the *d*-statistic and the set of variables included in our analytical framework. By carefully examining the impact of these variables on the *d*-statistic, we can gain further insights into the directional influence of macroeconomic factors on the *d*-statistic. This empirical analysis implies a pivotal role in advancing our comprehension of the fundamental structure underpinning the crisis theory of disproportionality. Additionally, the outcomes of this analytical inquiry empower us with the knowledge required to make informed assessments of the interactions among these variables.

Our main focus is on examining how the growth rate of fixed capital affects the *d*-statistic, as shown in Figure 4-1. The accumulative impulse response function reveals that a shock equivalent to an increase of one standard deviation in the growth rate of fixed capital has a positive and enduring impact on the *d*-statistic. This outcome aligns with our theoretical predictions and finds support in the works of various authors, including Hilferding and Maksakovsky, as previously discussed. Additionally, Clarke (2016) provides a lucid explanation of this phenomenon as follows: "*Every capitalist is forced by the pressure of competition to develop the productive forces without limit, and therefore is equally compelled to seek to expand the market by all means. But the more extensive is the market, the less can it be controlled, and so the more vulnerable is it to disruption as proportional relationships break down.*" (p. 229)

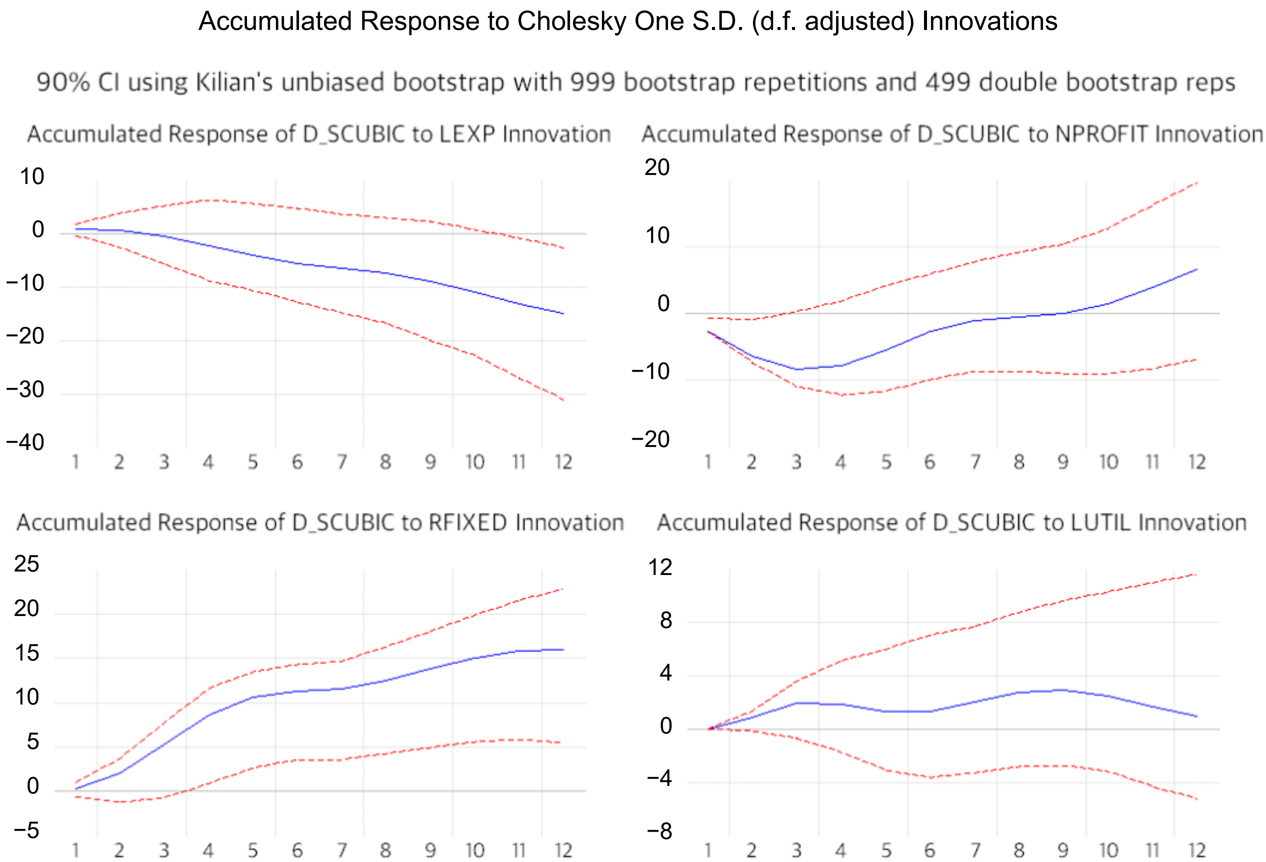
In essence, a faster expansion of total fixed capital implies a vulnerability in the proportional relationships essential for ensuring a stable process of reproduction over an extended duration.[18] Nonetheless, the allocation of production in alignment with a new and suitable ratio is a formidable task, primarily due to the disorganization inherent in capitalist production. Consequently, this imbalance intensifies, ultimately precipitating a crisis (Rieu, 2016). Additionally, as Ehara (2018) discussed, within the context of capitalist market dynamics, multi-layered production conditions – where diverse production setups coexist within the same industry due to uneven technological advancements and the persistence of fixed capital – intensify competition and heighten uncertainty for individual capital entities when making investment decisions. This layered structure widens the gap between the short-term volatility of market prices and the relative stability of production prices, leading to greater instability in price structures. Such instability complicates accurate

18 We can also refer to Tugan-Baranovsky's rationale, rooted in the theory of investment lags. He underlines the significance of fixed capital in explaining crises by considering the transformation of potential purchasing power, denoted as free capital (or surplus capital) and the actual time lag required for the complete construction of fixed capital. To elucidate further, free capital (*e.g.*, potential purchasing power such as bank deposits) accumulates during economic downturns due to limited investment opportunities, with a primary allocation towards fixed capital. Nevertheless, the process of constructing fixed capital requires a substantial duration intrinsically. When free capital is allocated to fixed capital, it is prone to generate excessive demand relative to available goods. However, by the time the fixed capital becomes operational, the reservoir of free capital starts to deplete, leading to overproduction (*i.e.*, surplus supply).

market pricing and increases the likelihood of breakdowns in market mechanisms. Ultimately, the multi-layered nature of production conditions, marked by technological disparities and persistent fixed capital, highlights how imbalances may drive crises within the market system.

Furthermore, a comprehensive investigation of the dynamic interrelationship between the profit rate and the *d*-statistic is essential. A long-term analysis of this relationship reveals a consistent trend: fluctuations in the profit rate negatively affect the *d*-statistic, particularly over a period of approximately 3 years. This phenomenon can be interpreted from a Marxian perspective, suggesting that a declining profit rate adversely affects overall economic vitality. This study indicates that when profit rates decline, disturbances emerge in the proportional interdependencies across production sectors, which are crucial for maintaining the reproducibility of an economy. Understanding the interplay between the profit rate and economic proportionality is essential for sustaining stability and mitigating disruptions (Clarke, 2016).

Figure 4-2: (Accumulated) response of *d*-statistic to impulse of other variables

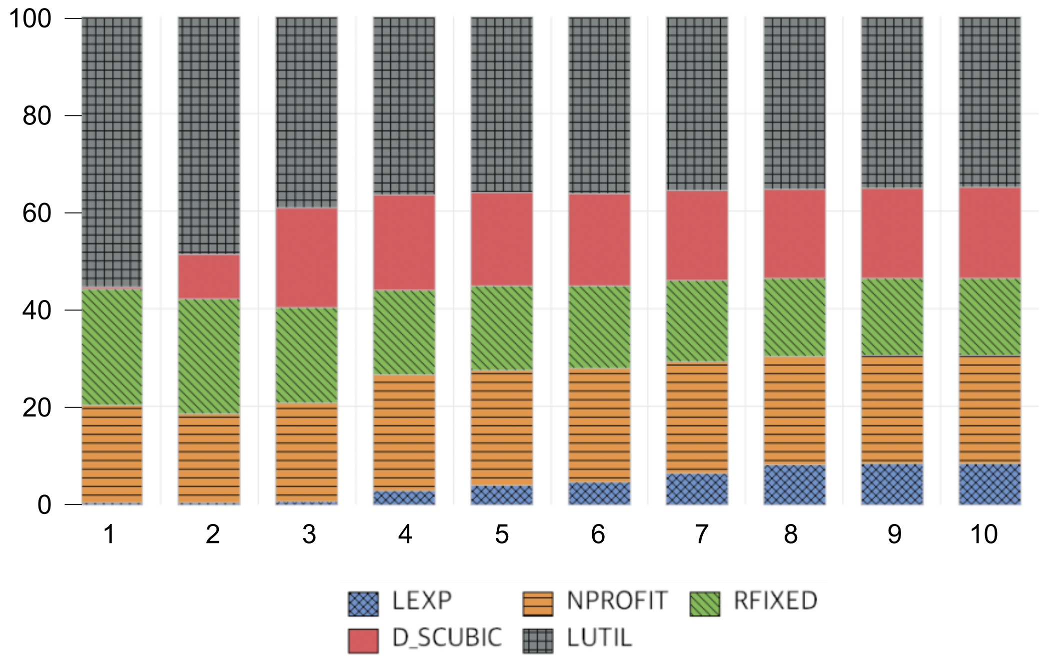


Note: 90% CI using Kilian's unbiased bootstrap with 999 bootstrap repetitions and 499 double bootstrap replications.

Source: Author's own elaboration

Figure 4-2 based on Kilian's unbiased bootstrap[19], on the other hand, offers an additional intriguing point with respect to the potential ramifications of augmented exports on a nation's disproportionality. Although the relationship is not statistically significant at 5%, a larger level of exports is associated with a decrease in the *d*-statistic in the long term (12^{th} periods) at 10% significance. This can be explained from two perspectives. Firstly, it can be inferred that exporting products facing overproduction may help alleviate economic disproportionality. Secondly, if the level of exports serves as a proxy for openness, increased capital mobility across borders could facilitate a faster and more efficient distribution of investment across sectors. This effect would appear as a decrease in the *d*-statistic. Consequently, it is reasonable to hypothesize that an increase in the export-to-GNI ratio may mitigate the risk of an economic crisis due to sectoral imbalances.

Figure 5-1: Results of variance decomposition – variance decomposition of log (utilization rate); using Cholesky (d.f. adjusted)



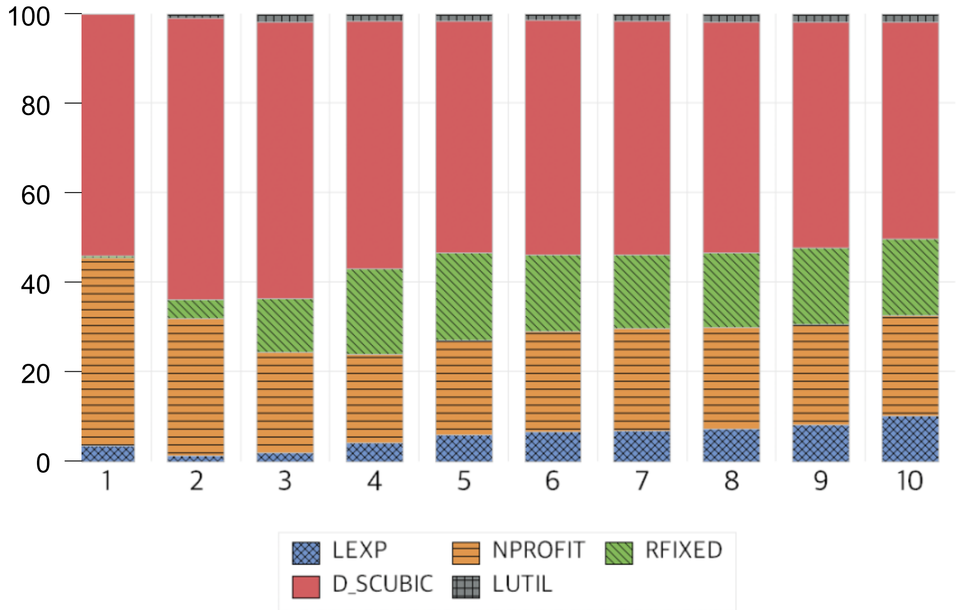
Source: Author's own elaboration

Lastly, we perform the variance decomposition, which quantifies the proportion of the forecast error variance associated with each variable attributed to external shocks of the other var-

19 It offers a method for obtaining bias-corrected confidence intervals which "*explicitly account for the bias and skewness of the small-sample distribution of the impulse response estimator*" (Kilian, 1998) when we use the VAR approach.

iables. Figures 5-1 and 5-2 convey that, starting from the 10th period onwards, approximately 18.77% of the fluctuations in the logarithmized utilization rate can be explained by variations in the *d*-statistic. Additionally, the proportion demands attention despite its relative modesty compared to other variables (excluding log (*exports*)). On the other hand, the variability in the growth rate of fixed capital accounts for approximately 17.19% of the *d*-statistic fluctuations beyond the 10th period, revealing the significance of comprehending the role of the growth rate of fixed capital in explaining the dynamics of overall disproportionality.

Figure 5-2: Results of variance decomposition – variance decomposition of *d*-statistic; using Cholesky (d.f. adjusted)



Source: Author's own elaboration

4. Concluding Remarks

We sought to empirically validate the disproportionality theory of crises, a longstanding subject of discussion in economics. Employing the Sraffian production price as our theoretical basis, we examined its implications. To gauge the disparity between Sraffian production prices and actual market prices, we advocated the use of a numeraire-free index derived from input-output (IO) tables, thus avoiding reliance on any arbitrary normalization conditions. Subsequently, we subjected Korean data to scrutiny using a structural vector autoregression (SVAR) model, and our findings lend support to our hypothesis that the deviation between Sraffian prices of production and market prices may lead to a crisis. In addition, our analysis highlights the critical role of the growth rate of fixed capital in explaining the relationship between sectoral disproportionality and economic activity – a conclusion consistent with the insights of eminent economists such as Maksakovsky, Hilferding and Tugan-Baranovsky.

In essence, this study demonstrates the importance of indicators such as the *d*-statistic in macroeconomic time series analysis, expanding their usage beyond their conventional role in cross-sectional analysis, where they estimate the ability of labour value or production prices to account for actual market prices. Furthermore, we corroborate that the concept of the Sraffian production price transcends abstract linear algebra, instead serving as a substantive reference point for comprehending real dynamics of an economy. Our research also elucidates that the degree of disproportionality at a given juncture can be quantified based on an available IO table, encompassing both market prices and given method of production, from which Sraffian production prices can be derived in principle. Consequently, this study serves as a crucial precursor for researchers exploring the theory of disproportionality crisis empirically. However, further rigorous investigations are warranted to determine whether similar outcomes emerge in the datasets of different countries, thereby providing additional support for the hypotheses that form the foundation of the theory of disproportionality.

Despite our contributions, there exist limitations necessitating attention in subsequent analyses. First and foremost, our empirical investigation overlooked variables associated with monetary factors, which are important in Tugan-Baranovsky's crisis theory, particularly the concept of "free capital". Secondly, our reliance on interpolated data due to the irregular publication of Korea's input-output tables since 1980 is another shortcoming. Furthermore, our study did not explicitly address cointegrations among variables, which requires rectification in future research. Thus, additional inquiries are imperative to compensate for these identified weaknesses.

Finally, our time series analysis raises the issue of whether institutional intervention is needed to reduce the deviation between Sraffian production prices and market prices to prevent crises. On the one hand, ensuring more efficient capital mobility through market mechanisms could progressively reduce these deviations over time. In particular, our findings suggest that variables such as an increase in the export ratio can help mitigate disproportionality, indicating that a more open economy may enhance capital mobility and thereby reduce structural imbalances. From a policy maker's perspective, encouraging openness and facilitating cross-border capital flows could serve as effective measures to address disproportionality.

On the other hand, the lack of market regulation may lead to the risk of excessive concentration of private investment in certain industries, which could heighten disproportionality. Therefore, promoting industrial diversification through public investment or fiscal policy emerges as another key recommendation from our study. Ultimately, our findings underscore the need for further research into the policy implications of managing disproportionality to build economic resilience.

The etymology of the term "crisis" (κρίσις), nonetheless, conveys a sense of judgement, often implying retribution for deviating from established legal norms. In this light, could recurrent economic crises be construed as judgements when prevailing market exchange rates deviate significantly from production prices, a reference point set by the state of production technology? Should the market system fail to autonomously realign actual exchange rates with the prescribed reference point, *i.e.*, the production price, it may necessitate proactive and associative institutional interventions surpassing laissez-faire principles to avert these recurrent "judgements".

"*In place of the principle of freedom based on individualism, a new great principle is emerging: association. It is the liberation of mankind from the catastrophe and misfortune that accompanies the development of industry today. The most important task of our time is social reform. But how will this task be realized? Through what combination can humanity succeed in reconciling individual freedom with the need for regulation of the social production process? All of this must determine the future*" (Tugan-Baranovsky, 2019, p. 146, translated by excerptor).

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Appendix

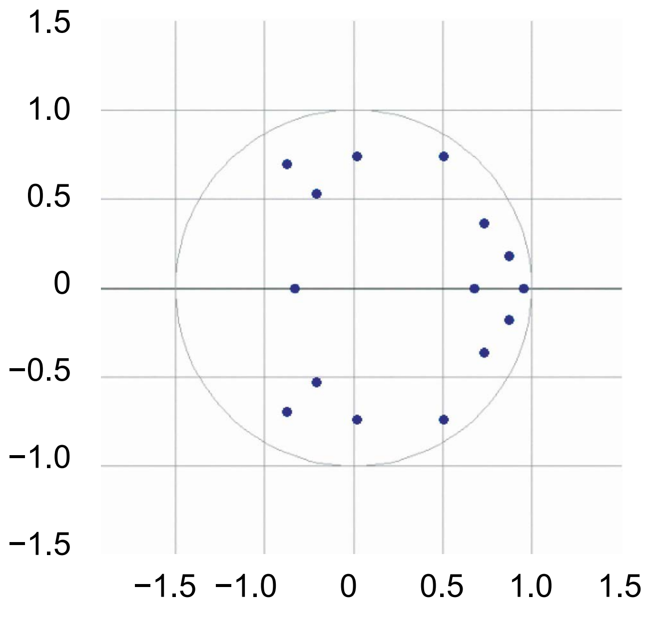
Table A1: VAR lag length selection criteria with five variables

Lag	LR	FPE	AIC	SIC	HQ
0	NA	0.126126	12.11884	12.33878	12.19561
1	251.5758	0.000117	5.121871	6.441470*	5.582446
2	42.95639	9.18e-05	4.792504	7.211769	5.636892
3	**52.29630***	**3.40e-05***	**3.566577***	7.085508	**4.794779***
4	18.39047	6.65e-05	3.729435	8.348032	5.341449

Notes: * indicates lag order selected by the criterion; LR: sequential modified LR test statistic (each test at 5% level); FPE: Final prediction error; AIC: Akaike information criterion; SIC: Schwarz information criterion; HQ: Hannan–Quinn information criterion.

Source: Author's own calculations

Figure A1: Inverse roots of AR characteristic polynomials



Source: Author's own elaboration