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Evaluation of Accuracy of Exchange Rate Expectation Models for Understanding Observed Expectations

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Abstract
Exchange rate expectations are a crucial element in the main monetary models. Therefore, this paper analyses the mechanism behind their formation. To achieve this, we analyse traditional expectation models using data from the Survey of Professional Forecasters (SPF) for the CZK/EUR currency pair. The data used cover one-year expectations in the period from January 2001 to December 2022, which are provided monthly by the Czech National Bank (CNB). The paper demonstrates the poor performance of the perfect expectation model. Furthermore, it demonstrates that traditional models, such as static, extrapolative, regressive and adaptive expectations, exhibit some explanatory power but lack robustness. The only traditional model that exhibits robustness is the model based on the UIP puzzle, which also outperforms all other traditional models when evaluated using error metrics. Based on these observations, the paper introduces a non-traditional model in which agents simply shift the current spot value by a constant into the future. This model displays robustness and outperforms the others.

JEL Classification: F31, E47, G17

Keywords: Financial markets; exchange rate; CZK/EUR; expectations

1. Introduction

Exchange rate expectations are crucial information for traders and central banks. They are a crucial component of the main monetary models for small open economies and an important part of analysing exchange rate fundamentals and behaviour. The exchange rate expectations are included, for example, in the DSGE model used by the Czech Ministry of Finance (Aliyev *et al.*, 2014) and the G3+ model used by the CNB (Brázdik *et al.*, 2020).

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This paper describes how expectations for the CZK/EUR pair are formed. To achieve this objective, the paper analyses various traditional expectation models and identifies the one that best aligns with the observed data. The paper uses a methodology that allows a reliable comparison between the models. To accomplish this, the traditional models are applied to the observed data of one-year expectations for the CZK/EUR pair, which are provided monthly in the Survey of Professional Forecasters (SPF) by the CNB. Optimal parameters are calculated, and the models are then compared using two error metrics: mean absolute error (MAE) and root mean square error (RMSE). Subsequently, the models undergo testing for stationarity. Following that, their first differences are analysed using linear regression.

Next, the models are tested for stability, and subsequently for robustness using the Mann–Whitney *U* test. The paper concludes that the most accurate traditional model across both error metrics is the one based on the UIP puzzle, indicating that agents regard interest rates as a significant variable when forming their expectations, but in the opposite way as expected by the relationship of uncovered interest rate parity. The analysis also shows that the traditional models provide informative insights when examined through linear regressions, as they all, except the perfect expectation model, show significance.

However, the paper shows that all traditional models, except the one based on the UIP puzzle, seem to exhibit bias, resulting in confirmed non-robustness when tested using the Mann–Whitney *U* test. Additionally, the paper demonstrates that agents tend to expect further depreciation of the CZK/EUR pair (stronger CZK) from its actual value. In response, the paper introduces a simple non-traditional model in which agents anticipate a consistently shifted actual value in the future. The paper shows that this model excels at capturing the observed data, as indicated by error metrics, showing the stability of this relationship and its robustness. While this model is solely quantitative, the paper delves into the underlying fundamentals. Nonetheless, it identifies this aspect as an open question warranting further research.

As an additional contribution to previous research into this currency pair (*e.g.*, Baghestani and Danila, 2014; Mandel and Vejmělek, 2021a; Mandel and Vejmělek, 2021b; Kladívko and Österholm, 2023), this paper employs a methodology that allows a reliable comparison of models, in addition to confirming or rejecting their significance. This paper also introduces an additional model for shifted static expectations and assesses the robustness and non-skewness of all the models when compared to observed data, using the Mann–Whitney *U* test. The comparison of the results with previous research by other authors is discussed in a separate section.

The paper is structured as follows: Section 2 provides a review of the work of other authors in this area. Section 3 describes the methodology used in this paper. Section 4 presents the analysed data and their basic characteristics. Section 5 deals with the empirical formation of exchange rate expectations, the optimal values of the coefficients and the performance of individual models. Section 6 interprets and discusses the results. Section 7 compares the results with the previous research by other authors on this currency pair. The last section summarizes the findings.

2. Previous Research

We can find long-term research into the formation of expectations for globally important exchange rates. Authors typically test exchange rate expectations for unbiasedness, construct various expectation models, analyse whether the models are significant, analyse risk premiums and look for differences between short-term and long-term expectations. Another common research method is to analyse how exchange rate expectations react to news and whether the reactions are rational or tend to overshoot.

The first common result from decades of this research is that exchange rate expectations are usually not rational (*e.g.*, Froot, 1990; Takagi, 1991; Dreger and Stadtmann, 2008). These results are based on tests of unbiasedness, non-orthogonality and non-rational reactions to news. The second common result from other authors is that exchange rate expectations have a significant backward-looking element (Takagi, 1991; Svendsen, 1993; Bénassy-Queré *et al*., 2003). The third common result is that exchange rate expectations are highly heterogeneous between agents (MacDonald and Taylor, 1992; Bénassy-Queré *et al*., 2003; Dreger and Stadtmann, 2008).

In the empirical part, this paper utilizes various expectation models that have evolved within economic literature over time. Throughout the twentieth century, it became evident that the formation of expectations in economic models has far-reaching consequences for their overall behaviour. One of the earliest and simplest expectation models was the static expectation model, in which agents anticipate the future value to remain the same as the present value. From a methodological perspective, this model holds some logical validity in the context of time series that are dynamically traded, such as exchange rates. This assumption is based on the notion that the anticipated future developments are already reflected in their current values.

The widely used concept of economic expectations was utilized by Cagan (1956) and Friedman (1968). It is referred to as adaptive expectations, wherein agents adjust their previous expectations while considering newly observed realizations of time series data. Another methodological approach to modelling expectations assumes extrapolative tendencies, where agents project past movements into the future. This concept is known as extrapolative expectations, originally developed by Metzler (1941) and has been employed in areas such as modelling of exchange rate expectations, as is employed, among others, in Froot (1990).

Another conceptual approach involves considering an equilibrium value towards which the time series tends to converge; this is referred to as regressive expectations. The paper by

Bénassy-Queré *et al*. (2003) employed a moving average as a filtering method to estimate the equilibrium value. Another simpler model is the perfect expectation model, where the anticipated value aligns with the already observed one. However, this model is only applicable ex-post, when subsequent realizations are observed, thereby diminishing its usefulness.

Another factor to consider when analysing exchange rate expectations is the impact of interest rates. These models suggest that expectations about future exchange rate moves are shaped by the interest rate difference between the two involved countries. One of the first research studies on this concept was conducted by Fischer (1930). Based on this work, the concept of the international Fischer effect was introduced. It posits that the currency of the country with a higher interest rate should be expected to depreciate, as the yield rates for individual countries should be expected to equalize after taking into account the current spot exchange rates. This relationship is called uncovered interest rate parity in modern literature.

However, the impact of higher interest rates on expectations of further currency movements is not entirely clear. Starting with the influential work of Fama (1984), the research usually indicates that agents expect the appreciation of the currency of the country with higher interest rates.[1] These observations, in contrast to the original international Fisher effect, are referred to as the UIP puzzle. One possible explanation for these observations is that higher interest rates encourage international capital inflows into the country, leading to an increased demand for the currency with the higher interest rate. Another factor that comes into play is the country-specific risk and global risk aversion, as analysed by Kalemli-Ozcan and Varela (2021).

The typical framework for understanding expectations is the rational expectation model. This concept suggests that agents form forward-looking expectations that are unbiased. This modelling approach originated with Muth (1961), and its properties have been extensively tested in the context of exchange rates. However, the rational expectation model is not included in this paper as the structure of the model does not allow a quantitative comparison with other models. At the same time, there is no unified methodology related to this model. Under some conditions, the rational expectation model can be expressed as a further analysed perfect expectation model, while under other conditions it can be also expressed as an analysed static expectation model. For these reasons, the model was omitted in this paper.[2]

1 This issue was also addressed by Froot and Thaler (1990). Among the recent papers, it has been studied by Candian and De Leo (2023) and Kremens *et al*. (2023). In the first mentioned recent paper, the authors also discuss the dynamic aspect of this issue, where the expected impact of the higher interest rate differs according to the time horizon for which the forecast is made. Backus *et al*. (2011) analysed the connection of these observations with monetary policy.

2 Nevertheless, Mandel and Vejmělek (2021a) rejected the testable aspects of the rational expectation model when applied to expectations for the CZK/EUR currency pair, based on tests of unbiasedness and serial correlation of forecasting errors.

From the perspective of the CZK/EUR pair, the primary focus of the overall research is on its equilibrium value, the determinants of the forward rate and the formation of risk premiums, rather than on the formation of expectations itself. However, the author found four papers that directly analyse CZK/EUR expectations.

Baghestani and Danila (2014) analysed time series data from a survey of exchange rate expectations conducted by the CNB. The period examined in the paper spanned from January 2005 to December 2012. Additionally, the authors divided their analysis into segments for domestic and foreign analysts, as well as various timeframes. The authors found that exchange rate expectations were inferior to a random walk and were directionally accurate. The authors also presented mixed results regarding the efficiency of these expectations and the presence of systematic bias in exchange rate forecasts.

Mandel and Vejmělek (2021a) analysed the formation of CZK/EUR expectations in the CNB's survey. The authors examined the period from May 1999 to December 2019, which they divided into two parts. The first period analysed was from May 1999 to December 2007, and the second period was from January 2008 to December 2019. The research found that expectations were not rational and provided mixed results for two other models. The paper confirmed adaptive expectations for monthly and annual forecasts after January 2008 but rejected them in the previous period. The extrapolative expectation model was confirmed at the 5% confidence level for the first period and at the 10% confidence level for the second period.

Mandel and Vejmělek (2021b) analysed the uncovered interest rate parity model. The paper examined the model ex-ante, utilizing observed exchange rate expectations (gathered from expert forecasts), and ex-post, based on the observed behaviour of the exchange rate. The authors conducted tests for short-term (one-month) and long-term (one-year) expectations. Regarding the ex-ante accuracy of the model for the long term, the descriptive statistics and correlation matrix did not provide clear findings regarding the validity of the international Fisher effect concerning the CZK/EUR pair. Cointegration analysis and VECM modelling indicated the following dynamic behaviour: in the first phase, the depreciation of the domestic currency leads to an increase in the interest rate, triggered by the response of the domestic central bank. In the second phase, the higher domestic interest rates induce depreciation expectations.

Kladívko and Österholm (2023) analysed the time series provided by the survey of expert forecasts from the CNB. The paper investigated whether the forecasts can outperform the random walk for both short-term and long-term horizons. Concerning the time series for the CZK/EUR exchange rate pair, the authors found that in the short term (one-month expectations), the forecasts did not outperform the random walk, while in the long term (one-year expectations), the forecasts did outperform the random walk. However, in neither the short term nor the long term did the analysis for CZK/EUR pair show results that were statistically significant.

Based on recent research into major global exchange rates, Candian and De Leo (2023) analysed the impact of interest rates on exchange rate expectations. The authors examined the differences between models based on interest rates, specifically the uncovered interest rate parity model and the UIP puzzle, and observed expectations. In their model, errors result from factors within monetary policy. Underreactions occur when agents overestimate the impact of temporary shocks, and overreactions happen when investors incorrectly perceive long-lasting shocks as more autocorrelated than they truly are. According to the model, the authors were able to explain the changes in deviations between observed exchange rate expectations and interest rate-based models that emerged after the global financial crisis.

Also, from the recent research into major global exchange rates, a different approach to the analysis was employed by Kremens *et al*. (2023). The authors analysed exchange rate expectations for various currencies based on macroeconomic fundamentals, such as GDP, the current account and the performance of equity markets, in contrast to traditional expectation models. The authors demonstrated that agents can successfully forecast appreciation over a two-year horizon. However, when controlling for the three aforementioned macroeconomic variables, survey expectations are not able to successfully forecast currency appreciation in the sample, suggesting that agents do not rely on any "secret" personal information.

3. Methodology

In the empirical part, we will employ a quantitative description of traditional expectation models. Since the paper analyses annual expectations, the operator $t + 12$ represents the time operator for the same period next year. The currency pair under analysis is CZK/EUR, with the EUR considered as the domestic currency. As a result, appreciation of the CZK/EUR pair indicates the weakening of the CZK, while depreciation indicates the strengthening of the CZK. We will denote $E_t(SR_{t+12})$ as the expected exchange rate predicted by the model and (SR_{t+12}) for the observed values of exchange rate expectations in the data. The notation [.] represents the closed interval for the coefficient value.

The first model used is the regressive expectation model, described in Equation (1):

$$E_t\left(SR_{t+12}\right) = \gamma SR_t + \left(1-\gamma\right)SR_t^* \quad \gamma \in \left[0,1\right] \tag{1}$$

where SR is the spot exchange rate, t is a time operator, E is an operator for expectations, and SR^* is the long-run equilibrium exchange rate. The long-term equilibrium value SR_t^* is estimated via the Hodrick–Prescott filter. The filter is computed with $\lambda = 14{,}400$, which is the optimal lambda for monthly data (as derived in Hodrick and Prescott, 1997). For an observed value y and trend value T, the filter is a minimization task as follows:

$$\min_T\Big(\sum_{t=1}^{T}(y_t - T_t)^2 + \sum_{t=2}^{T-1}\big((T_{t+1} - T_t) - (T_t - T_{t-1})\big)^2 \tag{2}$$

The second model used is the adaptive expectation model, in which agents use the actual exchange rate and the expected exchange rate from the previous period. The model is described by Equation (3), where the weight of the past expectations is represented by the coefficient $1 - \alpha$.

$$E_t(SR_{t+12}) = \alpha SR_t + (1 - \alpha)E_{t-12}(SR_t) \qquad \alpha \in [0,1] \tag{3}$$

Another model used is the extrapolative expectation model, where agents extrapolate past movements in the exchange rate into the future. This model is described by Equation (4), where the coefficient of extrapolative tendencies β can take values of any real number.

$$E_t(SR_{t+12}) = SR_t + \beta(SR_t - SR_{t-12}) \qquad \beta \in \mathbb{R} \tag{4}$$

The next model employs perfect expectations, as described in Equation (5) below:

$$E_t(SR_{t+12}) = SR_{t+12} \tag{5}$$

Another model uses the relationship of the expected movements in the exchange rate based on the differences between interest rates. The model is described by Equation (6), where IR_{EUR} is the one-year EURIBOR rate and IR_{CZK} is the one-year PRIBOR rate. As mentioned in Section 1, the direction of the reaction to the interest rate differential is ambiguous. To address this uncertainty, we introduce the parameter ω. A positive ω signifies that a higher domestic interest rate is expected to result in appreciation of domestic currency (the UIP puzzle), while a negative ω indicates that a higher domestic interest rate is expected to lead to depreciation of domestic exchange rate (the international Fischer effect). This relationship is formally expressed in Equation (6).

$$E_t\left(SR_{t+12}\right) = SR_t * \omega\left[\frac{1 + IR_{EUR,\,t}^{t+12}}{1 + IR_{CZK,\,t}^{t+12}}\right] \qquad \omega \in \mathbb{R} \tag{6}$$

Given that the empirical part demonstrates a positive value of the parameter ω in the CZK/EUR pair, we will refer to this model as the UIP puzzle from now on.

Another model uses static expectations, where the expected exchange rate is equal to the currently observed exchange rate, and can be described as follows:

$$E_t(SR_{t+12}) = SR_t \tag{7}$$

Up to this point, six traditional expectation models have been established. Now, we are introducing an additional non-traditional model. As will be demonstrated in Section 5, agents tend to base their expectations on the present spot rate and usually predict depreciation of the CZK/EUR pair (stronger CZK). This leads to the concept of refining the static expectation model by incorporating a shifting parameter, labelled as ξ. The non-traditional shifted static expectation model can be outlined as follows:

$$E_t(SR_{t+12}) = \xi SR_t \qquad \xi \in \mathbb{R}_+ \tag{8}$$

When analysing all the models, the model accuracy is calculated using two widely used ratios, the mean absolute error (MAE) and root mean square error (RMSE). Both of these metrics utilize the error (e), which is computed as the difference between the forecasted value by the model $E_t(SR_{t+12})$ and the observed data $E_t^O(SR_{t+12})$.

$$e_t = E_t\left(SR_{t+12}\right) - E_t^O\left(SR_{t+12}\right) \tag{9}$$

Then, the metrics are calculated according to Equations (10) and (11), where n represents the number of observations. The model accuracy ratios are computed for the theoretical model values and observed expectations in the data from January 2002 to December 2021 for all the models mentioned above. The data for the years 2001 and 2022 are used for estimating the models (more details in Section 3). I decided to use two ratios, each with its own specific characteristics, in order to avoid skewing the results by relying on a single ratio.

The metric of mean absolute error is described in Equation (10) and the root mean squared error in Equation (11). The results of the tests are then compared for all the expectation models.

$$MAE = \frac{1}{n}\sum |e_t| \qquad RMSE = \sqrt{\frac{1}{n}\sum e_t^{\,2}} \tag{10), (11}$$

The coefficients α, β, γ and ξ (which minimize the ratios of error metrics) were computed numerically. The numerical computation involves solving the minimization task to find the coefficients that minimize the error metric. From Equations (10) and (11), it is evident that both metrics have the same optimal coefficients.

For the regressive expectation model, the minimization task takes the following form, where both equations yield the same results:

$$\underset{\gamma\in[0,1]}{\operatorname{argmin}}\left\{\frac{1}{n}\sum \left|\gamma SR_t + (1-\gamma) SR_t^* - E_t^O\left(SR_{t+12}\right)\right|\right\} \tag{12}$$

$$\underset{\gamma\in[0,1]}{\operatorname{argmin}}\left\{\sqrt{\frac{1}{n}\sum\left[\gamma SR_t + (1-\gamma) SR_t^* - E_t^O\left(SR_{t+12}\right)\right]^2}\right\} \tag{13}$$

For the adaptive expectation model, the minimization task is as follows:

$$\underset{\alpha\in[0,1]}{\operatorname{argmin}}\left\{ \frac{1}{n}\sum\left| \alpha SR_t + (1-\alpha) E_{t-12}(SR_t) - E_t^O(SR_{t+12}) \right| \right\} \tag{14}$$

$$\underset{\alpha\in[0,1]}{\operatorname{argmin}}\left\{ \sqrt{\frac{1}{n}\sum\left[\alpha SR_t + (1-\alpha) E_{t-12}(SR_t) - E_t^O(SR_{t+12}) \right]^2} \right\} \tag{15}$$

For the extrapolative expectation model, the minimization equations are:

$$\underset{\beta\in\mathbb{R}}{\operatorname{argmin}}\left\{ \frac{1}{n}\sum\left| SR_t + \beta(SR_t - SR_{t-12}) - E_t^O(SR_{t+12}) \right| \right\} \tag{16}$$

$$\underset{\beta\in\mathbb{R}}{\operatorname{argmin}}\left\{ \sqrt{\frac{1}{n}\sum\left[SR_t + \beta(SR_t - SR_{t-12}) - E_t^O(SR_{t+12}) \right]^2} \right\} \tag{17}$$

For the UIP puzzle expectation model, the minimization task has the following form:

$$\underset{\omega\in\mathbb{R}}{\operatorname{argmin}}\left\{ \frac{1}{n}\sum\left| SR_t * \omega\left[\frac{1+IR_{EUR,\,t}^{t+12}}{1+IR_{CZK,\,t}^{t+12}} \right] - E_t^O(SR_{t+12}) \right| \right\} \tag{18}$$

$$\underset{\omega\in\mathbb{R}}{\operatorname{argmin}}\left\{ \sqrt{\frac{1}{n}\sum\left[SR_t * \omega\left[\frac{1+IR_{EUR,\,t}^{t+12}}{1+IR_{CZK,\,t}^{t+12}} \right] - E_t^O(SR_{t+12}) \right]^2} \right\} \tag{19}$$

Lastly, for the shifted static expectation model, the formula is as follows:

$$\underset{\xi\in\mathbb{R}_+}{\operatorname{argmin}}\left\{ \frac{1}{n}\sum\left| \quad SR_t - E_t^O(SR_{t+12}) \right| \right\} \tag{20}$$

$$\underset{\xi\in\mathbb{R}_+}{\operatorname{argmin}}\left\{ \sqrt{\frac{1}{n}\sum\left[\quad SR_t - E_t^O(SR_{t+12}) \right]^2} \right\} \tag{21}$$

Subsequently, a test for the stationarity of time series is performed for each model using the augmented Dickey–Fuller test. Since the tests in Section 5 declined a unit root for all the time series, their first differences are taken. The significance of individual models is then analysed on stationary data through linear regressions.

For analysing the stability of parameters in the models with associated parameters and assessing the overall stability of other models over time, another augmented Dickey–Fuller test is conducted on the differences between the model-predicted values and the observed values. Finally, to evaluate the robustness of the models, the Mann–Whitney U test is performed. This test assesses the null hypothesis that the probability distribution of model outcomes is equivalent to the probability distribution of the observed data.

4. Data

The data of annual expert forecasts are provided monthly by the CNB and are used for the analysis of observed CZK/EUR expectations $E_t^O(SR_{t+12})$. The monthly averages of CZK/EUR prices are also provided by the CNB. The data used are in the range from January 2001 to December 2022, as the first survey published on the website of the CNB is from January 2001.

Table 1: Characteristics of analysed time series (for comparison timeframe January 2002 – December 2021)

	Mean	Median	Maximum	Minimum	Std. dev	Obs.
SR_t	27.22	26.73	32.98	23.53	2.24	240
$E_t^O(SR_{t+12})$	26.61	25.89	31.93	23.68	2.19	240
$SR_t - SR_{t-12}$	−0.42	−0.35	3.08	−4.80	1.44	240
$E_{t-12}SR_t - SR_{t-12}$	−0.58	−0.55	1.03	−2.75	0.46	240
$(IR_{CZK,t}^{t+12}) \times 100$	1.88	1.83	4.51	0.43	1.16	240
$(IR_{EUR,t}^{t+12}) \times 100$	1.09	0.42	4.83	−0.58	1.58	240

Source: Author's own calculations, based on CNB (2023a), CNB (2023b), CNB (2023c) and ECB (2023)

Even though the data used are from January 2001 to December 2022, both forecast accuracy ratios and their optimal coefficients are computed for the range of January 2002 to December 2021. This is because the structure of two of the models requires the first year for estimation, and one model cannot be estimated for the last year[3]. Therefore, the first and last years are only used for model estimation and not for comparison between models. This approach of using one timeframe allows a reliable comparison between the models.

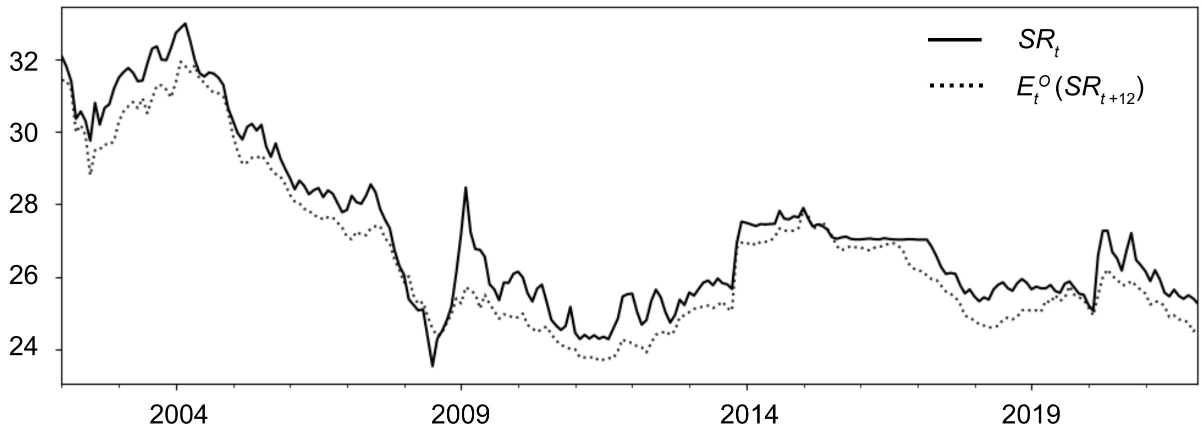
The data for PRIBOR are obtained from the database of the CNB, while the data for EURIBOR are obtained from the database of the ECB. The Python programming language is used for data analysis, numerical computations and visualization.

3 The adaptive and extrapolative expectation models cannot be estimated for the first year and need it for its estimation, and the perfect forecast model cannot be estimated for the last year.

5. Empirical Part

Before any computation, Figures 1 and 2 enable a visual analysis of the exchange rate SR_t and the observed exchange rate expectations $E_t^O(SR_{t+12})$. This analysis shows interesting outcomes. Firstly, it is obvious that the observed expectations $E_t^O(SR_{t+12})$ tend to copy the spot exchange rate SR_t, rather than forecasting future developments (Figures 1 and 2). Secondly, the observed expectations $E_t^O(SR_{t+12})$ are usually lower than the spot exchange rate SR_t, indicating that agents are expecting further depreciation (stronger CZK) nearly all the time (Figures 1 and 2). Thirdly, the volatility of the expected change $E_{t-12}(SR_t) - SR_{t-12}$ is lower than the later really realized change $SR_t - SR_{t-12}$ (Figure 2 and Table 1). Mandel and Vejmělek (2021a) presented the idea that analysts tend to avoid predicting significant changes in exchange rates due to the risk of high errors in their predictions.

Figure 1: Development of CZK/EUR expectations and actual exchange rate

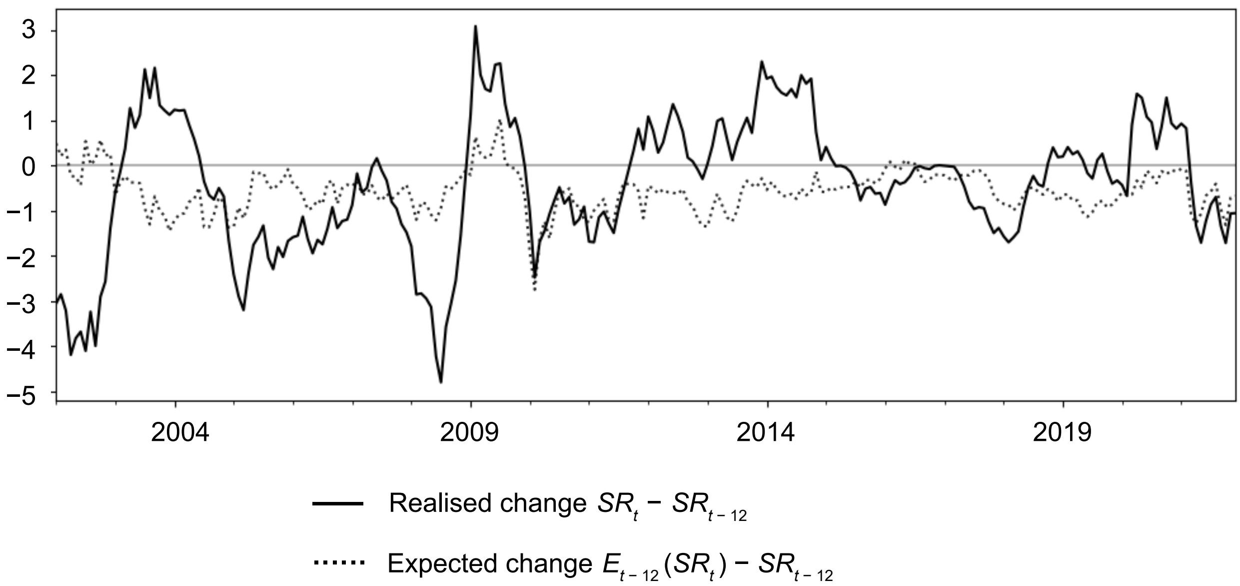


Note: The figure shows the spot exchange rate SR_t, and the observed expected future exchange rate $E_t^0(SR_{t+12})$.
Source: Author's own elaboration; based on CNB (2023a) and CNB (2023b)

In Figure 2, it is evident that agents consistently anticipated depreciation (stronger CZK), as indicated by the variable of expected change $E_{t-12}(SR_t) - SR_{t-12}$ being lower than zero. This anticipated depreciation (stronger CZK) was expected on 95% of the observed days. Additionally, Figure 2 shows that agents anticipated appreciation (weaker CZK) only twice during the analysed period: in 2002 and 2009 (as indicated by the variable $E_{t-12}(SR_t) - SR_{t-12}$ being higher than zero).

The numerically calculated optimal coefficients that minimize errors (e) in forecast accuracy ratios are presented in Table 2. Using these optimal coefficients, the final model accuracy ratios were computed and listed in Table 3, which presents the results. As shown in Table 2, all the estimated optimal coefficients have a non-zero value. From the perspective of individual models, the coefficient of extrapolative expectations has a lower value, indicating that the extrapolative part is not particularly important. Thus, it is expected that the efficiency of this model will be similar, but slightly better than that of the static expectations. The coefficient ω in the model based on interest rates has a positive sign, which shows that agents expect an appreciation of the currency with a higher interest rate (the UIP puzzle). The coefficient ξ for shifted static expectations has a value lower than 1, confirming the tendency of agents to forecast weaker levels of the CZK/EUR rate (stronger CZK) compared to its actual values. This confirms the previous observation that the expected change $E_{t-12}(SR_t) - SR_{t-12}$ is lower than zero for almost the whole analysed period.

Figure 2: Development of realised and expected change in CZK/EUR pair



Notes: The figure shows the realized change in the exchange rate $SR_t - SR_{t-12}$, while values higher than 0 indicate appreciation of the CZK/EUR pair (weaker CZK), and values lower than 0 indicate depreciation (stronger CZK). The figure also displays the expected change in the exchange rate $E_{t-12}(SR_t) - SR_{t-12}$ and compares the two.
Source: CNB (2023a) and CNB (2023b)

The stability of the models was assessed using the augmented Dickey–Fuller test, conducted on the time series of differences between the model and observed reality. For models with parameters, it indicates whether those parameters remain stable. For other models, it assesses their overall stability over time. The results of this analysis are presented in Table 7 in Appendix 4. The results demonstrate that all the models exhibit overall stability. Among the models with parameters, the adaptive, regressive and shifted static expectation models show parameter stability at the 1% significance level. Additionally, the extrapolative expectation model shows stability at the 5% significance level. Among the models that have no parameters, the static expectation model displays stability at the 1% significance level, while the perfect expectation model shows stability at the 5% significance level.

Table 3 presents the performance of the analysed models based on the error metrics. Among the traditional models, the model with perfect expectations exhibited notably worse performance, whereas the models with static and extrapolative expectations showed better results. As shown in the table, the third-best performance among the traditional models was achieved by the model with regressive expectations. It was followed by the model with adaptive expectations in the second position. In terms of the RMSE metric, the second-best result was shown by the model with extrapolative expectations. The best results among the traditional models were achieved by the model based on the UIP puzzle. Better results than the traditional models were obtained by the non-traditional shifted static expectation model. This model showed better results for both forecast accuracy ratios employed in this paper.

The Augmented Dickey–Fuller tests revealed that the time series for the models were non-stationary, except for the extrapolative expectations, which showed stationarity at a 5% *p*-value level, and the perfect expectation model, which exhibited stationarity at a 10% *p*-value level, as shown in Table 4 in Appendix 1. Consequently, we took the first differences, resulting in stationarity for all the time series, as demonstrated in Table 5 in Appendix 2. Subsequently, linear regressions were conducted on these stationary data, indicating that all the backward-looking and static models were statistically significant in explaining the observed data at a 1% *p*-value level. However, the forward-looking perfect expectation model did not exhibit significance. The detailed results of the linear regressions are shown in Table 6 in Appendix 3.

Table 2: Summarization of computed optimal coefficients

Model	Optimal coefficient
Adaptive expectations	$\alpha = 0.788$
Extrapolative expectations	$\beta = 0.133$
Regressive expectations	$\gamma = 0.603$
UIP puzzle expectations	$\omega = 0.985$
Shifted static expectations	$\xi = 0.977$

Note: The coefficients that minimize the error metrics are used.

Source: Author's own elaboration; based on CNB (2023a), CNB (2023b), CNB (2023c) and ECB (2023)

Table 3: Summarization of computed error metrics for various models

Model	MAE	RMSE
Perfect expectations	0.983	1.402
Static expectations	0.644	1.135
Extrapolative expectations	0.628	1.093
Regressive expectations	0.626	1.119
Adaptive expectations	0.606	1.101
UIP puzzle expectations	0.353	0.841
Shifted static expectations	0.312	0.791

Source: Author's own elaboration; based on CNB (2023a), CNB (2023b), CNB (2023c) and ECB (2023)

Despite linear regression demonstrating statistical significance in all the models except the perfect expectation model, Table 8 in Appendix 5 presents an analysis of the model robustness using the Mann–Whitney *U* test. The results indicate that only two models display robustness: the traditional UIP puzzle expectation model and the non-traditional shifted static expectation model. These findings can be attributed to the bias exhibited by the remaining backward-looking and static models. Model values indicate higher expectation levels (weaker expected CZK) than the expectations observed in the data, as shown in Figures 5, 6, 7 and 8 in Appendix 6.

Worth mentioning is that the CNB initiated an exchange rate commitment during the period from November 2013 to April 2017. Throughout this period, the central bank announced its commitment to prevent the CZK/EUR pair from depreciating (strengthening CZK) below the level of 27.00, aiming to avert the risk of the economy entering deflation. I choose not to incorporate any additional measures related to this period into the empirical analysis.

The rationale for this decision is based on the introduced ADF test, which analyses the differences between model and observed values, evaluating the stability of the models and their parameters. This test demonstrates the stability of all the models at a 1% significance level across the entire analysed period (as shown in Table 7 in Appendix 4)[4]. This indicates that formation of expectations during the commitment period does not differ significantly from other periods. Even during this specific period, as illustrated in Figures 1 and 2, agents anticipate further depreciation (stronger CZK), likely due to the absence of a specific duration announcement by the CNB regarding that commitment. Examination of all the backward-looking and static models

4 Except the perfect expectation model.

(Figures 3, 4 and Figures 5, 6, 7 and 8 in Appendix 6) reveals that the expectation models effectively capture the development of expectations during this period. The disparities observed during the commitment period are not greater than those observed throughout the entire analysed period.

6. Discussion

The perfect expectation model performed the worst in terms of all the error metrics. Moreover, it was the only model that did not demonstrate significance when analysed with linear regression. As can be seen in Figure 7 in Appendix 6, the perfect expectation model struggles with agents' inability to predict important exogenous events that have a significant impact on exchange rate behaviour (such as financial crises, central bank interventions, or the COVID-19 pandemic). As a result, the ability of the model to explain formation of expectations is low, given the high frequency of these unexpected exogenous events in the CZK/EUR pair.

All the other traditional models demonstrated better results, and they all showed significance in explaining the observed data at the 1% *p*-value level with the linear regression. It is quite interesting to note the relatively good performance of the simple static expectation model, as it suggests that the forecasting behaviour of agents can be explained by such a straightforward rule. The positive outcomes of the results for adaptive, extrapolative and regressive expectations are reflected in the non-zero values of the coefficients (α, β, γ), which implies that these parts of the models have explanatory value.

Among traditional models, the most favourable results are demonstrated by the expectation model based on the UIP puzzle. This outcome suggests that, when making forecasts, agents consider interest rate differentials as an important factor influencing future CZK/EUR behaviour. The connection between interest rates and exchange rates is a presumption in standard models, such as the Dornbusch model or the Mundell–Fleming model.[5] However, the expected relationship between interest rates and future exchange rates contradicts the original international Fisher effect, as the analysis shows that agents anticipate the currency of the country with a higher interest rate to appreciate. The UIP puzzle expectation model also exhibits stability, as demonstrated in Table 7 in Appendix 4. Moreover, it is the only traditional model displaying robustness, as assessed by the Mann–Whitney *U* test (Table 8 in Appendix 5).

Interestingly, the non-traditional shifted static expectation model outperforms the traditional ones. This model surpasses the performance of traditional expectation models according to the error metrics and exhibits robustness when subjected to the Mann–Whitney *U* test. Although it is not commonly used in the literature, this model accurately captures the observed patterns in CZK/EUR expectation formation. This confirms the previous observation in Figure 1. Noteworthy is the enduring stability of both the relationship and the coefficient ξ over time, as demonstrated in Table 7

5 Dornbusch (1976) and Mundell (1963).

in Appendix 4. However, the question why this model excels remains unanswered, as there is no direct or intuitive explanation.

One of the possible explanations is that it is simply the result of chance; however, the observed stability over time contradicts this hypothesis. Another potential explanation is that we have identified the model that agents use when forming their expectations. The inclination to anticipate depreciation (stronger CZK) in the future, as opposed to actual values, could stem from the fundamental models they employ. It is feasible that agents are generally optimistic, considering the Czech economy's convergence with the Eurozone. Therefore, in the ongoing process of convergence through the exchange rate channel, one should anticipate further depreciation (stronger CZK). However, the Balassa–Samuelson theorem expects rather strengthening of the currency of the country to which it is converging[6].

There is also an exotic possible explanation that this pattern is a direct observation of the peso effect[7]. However, this explanation seems highly unlikely in this case. It can be assumed that the expected rare disaster would be a situation that leads to a strong appreciation of CZK/EUR (weaker CZK), as this exchange pair is negatively sensitive to risk sentiment (risk-off sentiment of markets leads to an appreciation of this pair (weaker CZK), while risk-on sentiment leads to depreciation (stronger CZK)). It is difficult to imagine that agents would expect a rare disaster that would lead to extreme depreciation (stronger CZK). Despite that, observed expectations are skewed towards depreciation (stronger CZK), which argues against the presence of the peso effect in these data.

Although the reason for the performance of the simple static rule of shifted static expectations remains unclear, it provides a good description of the formation of the CZK/EUR exchange rate. The paper also showed good performance of other static and backward-looking rules. Although a strong element of backward-looking and static tendencies in exchange rate expectations is commonly found in studies by other authors as well (*e.g.*, Froot, 1990; Bénassy-Queré *et al.*, 2003), it is important to note that this observation does not necessarily imply that agents do not consider estimated future developments. It can be assumed that the estimated future developments are reflected in the current price of the exchange rate, as the market is dynamically pricing changes in the expectations.

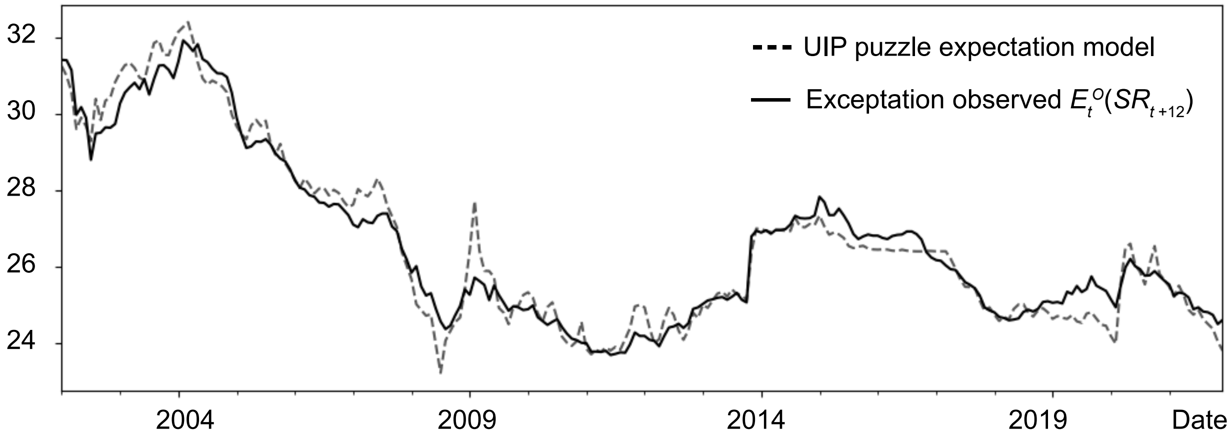
6 The theorem itself was analysed by Chen *et al*. (2015).

7 The peso effect is observed when there is a likelihood of a rare event that could significantly affect the forecasted value. This occurrence causes a permanent skew in the forecast. In such situations, the expectations take a specific form:

$$E_t(SR_{t+12}) = (1 - p_t)\, E_t(SR^0) + p_t E_t(SR^1)$$

where $E_t(SR^0)$ represents a standard forecast, $E_t(SR^1)$ is the forecast for the situation when the rare event happens, and p_t is the probability of SR^0 assessed by the market. More information can be found in Lizondo (1983).

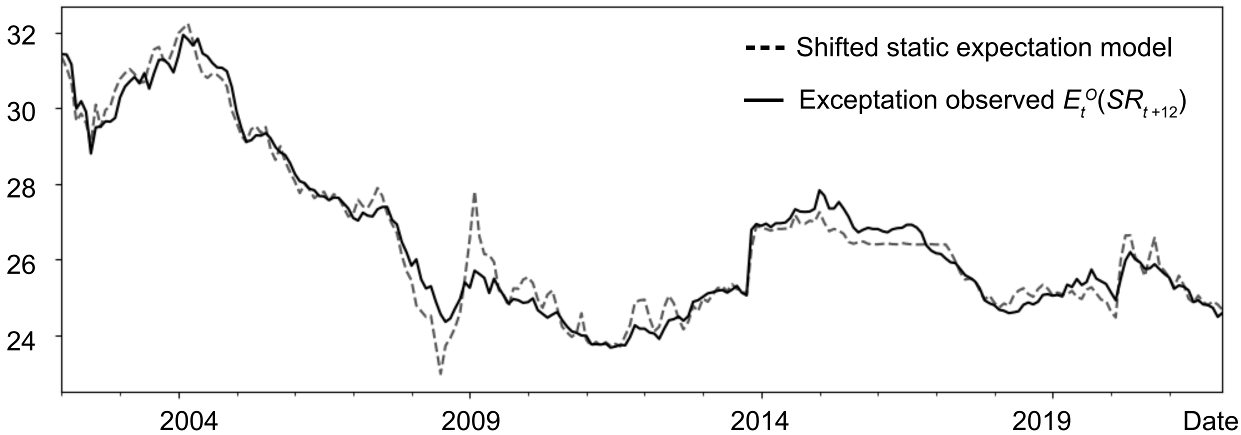
Figure 3: Comparison of observed and theoretical data for UIP puzzle expectation model



Note: The figure displays the expectation level indicated by the model based on the UIP puzzle, along with the expectations observed in the data $E_t^O(SR_{t+12})$.

Source: Author's own elaboration; based on CNB (2023a), CNB (2023b), CNB (2023c) and ECB (2023)

Figure 4: Comparison of observed and theoretical data for shifted static expectation model[8]



Note: The figure displays the expectation level indicated by the shifted static expectation model, along with the expectations observed in the data $E_t^O(SR_{t+12})$.

Source: Author's own elaboration; based on CNB (2023a) and CNB (2023b)

Expectations may therefore appear to be backward-looking or static, but they include adjustment of market prices based on the ongoing evolution of forecasts in the current value of the exchange rate. This fact represents a difference in analysing expectations of a variable that

8 Charts for all the remaining models are in Appendix 6.

is publicly traded with high liquidity (such as exchange rates), and thus represents a difference compared to analysing expectations of inflation or GDP.

The paper also presents high potential for further research. Firstly, it would be interesting to observe whether the non-traditional shifted static expectation model will continue to exhibit the highest accuracy in the future, and whether the coefficient ξ will remain stable. Another area with good potential for further research is to investigate whether the same coefficient values of the analysed models can be observed in other Central European currencies and other developing currencies around the world.

Additionally, it would be worthwhile to examine whether the same rules produce similar results when using other sources of expectations such as Refinitiv forecasts, forecasts of financial institutions and others. Finally, expanding the analysis to other time series provided by the CNB survey, such as inflation and interest rates, would also be a promising area of research. This could reveal whether expectations for other indicators behave in the same manner as those for the exchange rate.

7. Results Compared to Previous Research

In this section, we will compare our results with previous research into the CZK/EUR pair. The first paper we will reference is Mandel and Vejmělek (2021a). The paper also examined one -year exchange rate expectations, specifically focusing on extrapolative and adaptive expectations for the period from June 1999 to December 2019. In the case of extrapolative and adaptive expectations, the referenced paper employed a different methodology for estimation, utilizing linear regression and significance tests of the parameters.

The results in Mandel and Vejmělek (2021a) for the extrapolative expectation model indicate that the parameter β has a negative value, suggesting that agents expect depreciation following a period of previous appreciation and vice versa. Our paper, which uses a different timeframe and estimation methodology, shows a positive value for the parameter β, indicating the opposite relationship – an expected appreciation after a previous period of appreciation and vice versa. However, when compared to the other models in our paper, the parameter β exhibits a low value, indicating the limited added value of the extrapolative component of the model. Furthermore, an analysis of the robustness of the model in our paper demonstrates non-robustness, showing that the model does not effectively explain the observed data.

When comparing the results for adaptive expectation model, Mandel and Vejmělek (2021a) confirmed the significance of the model for the period from January 2008 to December 2019 but rejected its significance for the remaining period. With the different methodology and timeframe in this paper, we show that the model is significant when analysed for our entire period (January 2002 to December 2021) when subjected to linear regression. However, it does not exhibit robustness when tested using the Mann–Whitney U test. We also show that the two mentioned models are not the best-performing traditional ones, as they are outperformed by the expectation model based on the UIP puzzle.

Another referenced paper, Mandel and Vejmělek (2021b), presented conclusions that align with those revealed by our analysis. Using descriptive statistics, the authors demonstrated a relationship in which the currency of the country with a higher interest rate is expected to appreciate. This discovery led to the rejection of the ex-ante variant of uncovered interest rate parity. As a contribution in our paper, we quantified this relationship within the framework of the UIP puzzle model. One notable difference is that the aforementioned paper examined the dynamic aspects of this relationship using cointegration methods and the error correction model, which are not included in our paper due to differences in methodology.

Baghestani and Danila (2014) and Kladívko and Österholm (2023) differ from our paper in that the authors analysed the relationship between expectations and subsequent realized spot values of the exchange rate, while our research focuses on the formation of expectations themselves. Nevertheless, the aforementioned papers shared a common theme, which revolves around comparing the effectiveness of expectations to a random walk. Both mentioned studies concluded that one-year effectiveness of expectations outperforms the random walk, with the random walk expectations aligning with the static expectation model in our paper. Our conclusion is that the static expectation model underperforms other models, with the exception of the perfect expectation model. This finding suggests that agents take additional factors into consideration when forming their forecasts. Consequently, their expectations differ from those of the random walk model. The outcomes of the mentioned papers confirm the added value of these factors in forecasting future market movements.

8. Conclusion

This paper described how agents form their one-year exchange rate expectations for the CZK/EUR pair. To accomplish this, a range of economic expectation models was employed to identify the one that most accurately characterizes the data obtained from a monthly survey of professional forecasters conducted by the CNB. The contribution of the paper lies in the comprehensive analysis of different models, the methodology enabling direct quantitative comparisons, the quantification of the observed tendency among agents to forecast depreciation (stronger CZK) in this currency pair and an analysis of robustness of the models.

The paper demonstrated that among traditional models, the UIP puzzle expectation model, in which agents expect the appreciation of the currency of the country with a higher interest rate and vice versa, performs the best. Additionally, the paper introduced a non-traditional model called the shifted static expectation model, where agents anticipate that the current spot value of the exchange rate will persist into the future, shifted by a constant. The paper provides evidence that this non-traditional model outperforms all traditional models.

As said before, the paper showed that the traditional UIP puzzle expectation model outperforms other traditional models when its performance is evaluated using error metrics. Additionally, it is the only traditional model that demonstrates robustness when subjected to the Mann–Whitney *U* test. A stability test confirmed stability of the model over the entire analysed period. The strong performance of this model supports the prevailing view that agents consider interest rates as a fundamental factor in predicting future exchange rate movements. However, this result contradicts the uncovered interest rate parity theory and the related international Fisher effect, as the UIP puzzle model builds on the expectation that the currency of the country with a higher interest rate is going to strengthen (opposite to what the uncovered interest rate parity model expects). This finding is consistent with observations for other currency pairs and aligns with prior research into uncovered interest rate parity conducted by other authors.

The proposed best-performing non-traditional model, based on shifted static expectations, outperforms all the other models in terms of error metrics and demonstrates robustness when analysed using the Mann–Whitney *U* test. The test also confirms the model stability. Despite the discussion in this paper about the reasons behind the strong performance of this model, a definitive explanation has not been found, which highlights the potential for further research in this area.

The remaining analysed traditional adaptive, extrapolative, regressive and static expectation models showed stability when compared to the observed data. The model with extrapolative expectations exhibited stability at the 5% significance level, while the other models displayed stability at the 1% significance level. This indicates that these models remained consistent throughout the whole analysed period and did not exhibit time-varying behaviour. When analysed using linear regression, these traditional models also showed significance in explaining the observed data, with a *p*-value of 1%. Nevertheless, these models did not exhibit robustness when subjected to the Mann–Whitney *U* test. This discrepancy can be attributed to a consistent bias in predictions of those models, which consistently predict stronger values for CZK/EUR expectations (weaker CZK expectations) than what is observed in the data.

The last traditional model analysed in this paper is the perfect expectation model. The analysis revealed that the perfect expectation model performed poorly when evaluated using error metrics. It was also the only model that did not achieve statistical significance in the linear regression analysis. This suggests that agents are unable to generate perfect or nearly perfect forecasts for the future, which is expected due to the stochastic nature of exchange rates.

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Appendix 1

Table 4: Results of ADF test

Variable	Test statistic	*p*-value
Observed expectations I(0)	–2.0394	0.2696
Static expectations I(0)	–1.9043	0.3300
Adaptive expectations I(0)	–2.0994	0.2447
Extrapolative expectations I(0)	–3.3130	0.0143**
Regressive expectations I(0)	–2.6149	0.0900***
Perfect expectations I(0)	–2.0754	0.2545
UIP puzzle expectations I(0)	–1.6102	0.4779
Shifted static expectations I(0)	–1.9043	0.3300

Notes: *, **, *** denote that the test rejects the existence of a unit root at a 1%, 5% and 10% significance level, respectively.

Source: Author’s own calculations, based on CNB (2023a), CNB (2023b), CNB (2023c) and ECB (2023)

Appendix 2

Table 5: Results of ADF test on data after first difference

Variable	Test statistic	*p*-value
Observed expectations I(1)	–13.5755	0.000*
Static expectations I(1)	–10.4900	0.000*
Adaptive expectations I(1)	–10.5254	0.000*
Extrapolative expectations I(1)	–10.3007	0.000*
Regressive expectations I(1)	–10.8003	0.000*
Perfect expectations I(1)	–11.0291	0.000*
UIP puzzle expectations I(1)	–10.7631	0.000*
Shifted static expectations I(1)	–10.4900	0.000*

Notes: *, **, *** denote that the test rejects the existence of a unit root at a 1%, 5% and 10% significance level, respectively.

Source: Author's own calculations, based on CNB (2023a), CNB (2023b), CNB (2023c) and ECB (2023)

Appendix 3

Table 6: Results of linear regression

Dependent variable	Variable	Coefficient	Std. Error	*t*-statistics	Prob.	Stat.
Observed expectations I(1)	Static expectations I(1)	–0.0164 0.4242	0.013 0.037	–1.248 11.455	0.213 **0.000***	R^2 = 0.356 *F*-stat = 131.2 DW = 2.069
Observed expectations I(1)	Adaptive expectations I(1)	–0.0126 0.5076	0.013 0.046	–0.942 10.983	0.347 **0.000***	R^2 = 0.337 *F*-stat = 120.6 DW = 2.058
Observed expectations I(1)	Extrapolative expectations I(1)	–0.0184 0.3684	0.013 0.033	–1.393 11.248	0.165 **0.000***	R^2 = 0.348 *F*-stat = 126.5 DW = 2.072
Observed expectations I(1)	Regressive expectations I(1)	–0.0214 0.2099	0.013 0.019	–1.608 11.067	0.109 **0.000***	R^2 = 0.341 *F*-stat = 122.5 DW = 2.018
Observed expectations I(1)	Perfect expectations I(1)	–0.0263 0.0748	0.016 0.048	–1.609 1.566	0.109 0.119	R^2 = 0.010 *F*-stat = 2.453 DW = 1.746
Observed expectations I(1)	UIP puzzle expectations I(1)	–0.0157 0.4127	0.013 0.038	–1.171 10.965	0.243 **0.000***	R^2 = 0.337 *F*-stat = 120.2 DW = 2.032
Observed expectations I(1)	Shifted static expectations I(1)	–0.0164 0.4342	0.013 0.038	–1.248 11.455	0.213 **0.000***	R^2 = 0.356 *F*-stat = 131.2 DW = 2.069

Notes: *, **, *** shows statistical significance of a coefficient at a 1%, 5% and 10% significance level, respectively.
Source: Author's own calculations, based on CNB (2023a), CNB (2023b), CNB (2023c) and ECB (2023)

Appendix 4

Table 7: ADF test for difference between models and observed reality

Variable	Test statistic	*p*-value
Static expectations	−5.031	0.000*
Adaptive expectations	−3.902	0.002*
Extrapolative expectations	−3.213	0.019**
Regressive expectations	−3.604	0.006*
Perfect expectations	−3.270	0.016**
UIP puzzle expectations	−4.769	0.000*
Shifted static expectations	−5.012	0.000*

Notes: *, **, *** denote that the test rejects the existence of a unit root at a 1%, 5% and 10% significance level, respectively. The test of stationarity is conducted on the time series of differences between the model values and the observed values. The confirmed rejection of the unit root implies the stationarity of the differences between the model and observed data, thereby ensuring the stability of the model (and its coefficient) over the entire analysed period.

Source: Author's own calculations, based on CNB (2023a), CNB (2023b), CNB (2023c) and ECB (2023)

Appendix 5

Table 8: Mann–Whitney *U* test results

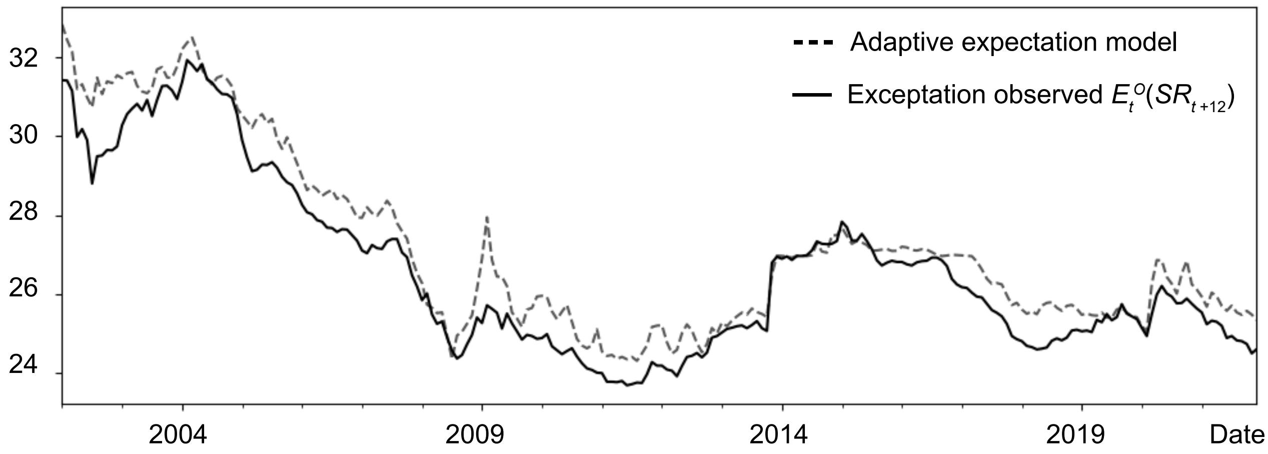
Variable	Test statistic	*p*-value
Static expectations	34,800	0.000*
Adaptive expectations	34,318	0.000*
Extrapolative expectations	23,434	0.000*
Regressive expectations	35,156	0.000*
Perfect expectations	32,545	0.014**
UIP puzzle expectations	28,099	0.645
Shifted static expectations	28,694	0.945

Notes: The Mann–Whitney *U* test analyses the robustness of the model in explaining observed data. *, **, *** denote that the test rejects the model robustness at the 1%, 5% and 10% significance levels, respectively. The rejection implies that the probability distribution of model outcomes differs from that of the observed data, indicating a lack of model robustness.

Source: Author's own calculations, based on CNB (2023a), CNB (2023b), CNB (2023c) and ECB (2023)

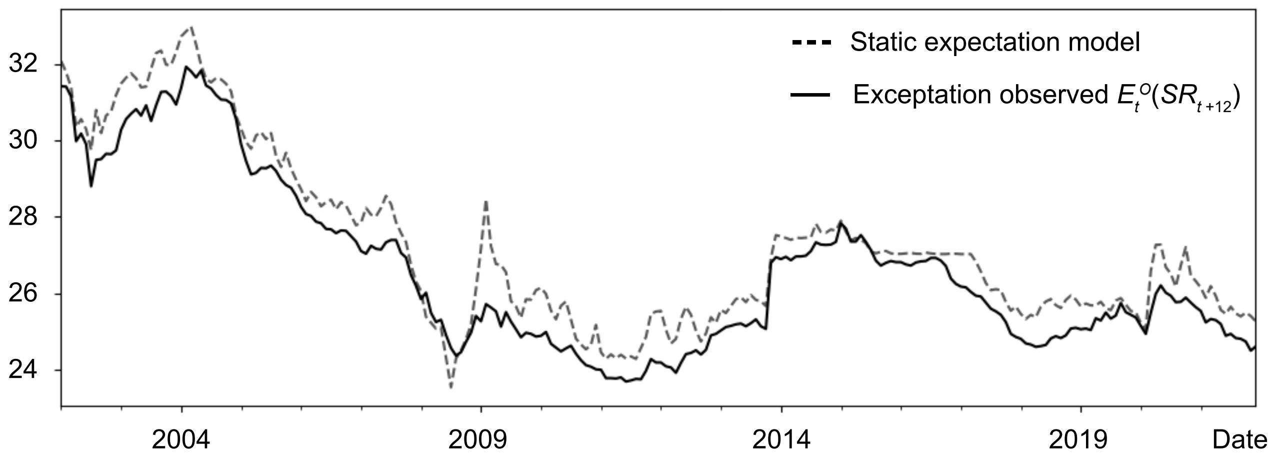
Appendix 6

Figure 5: Comparison of observed and theoretical data for adaptive expectation model



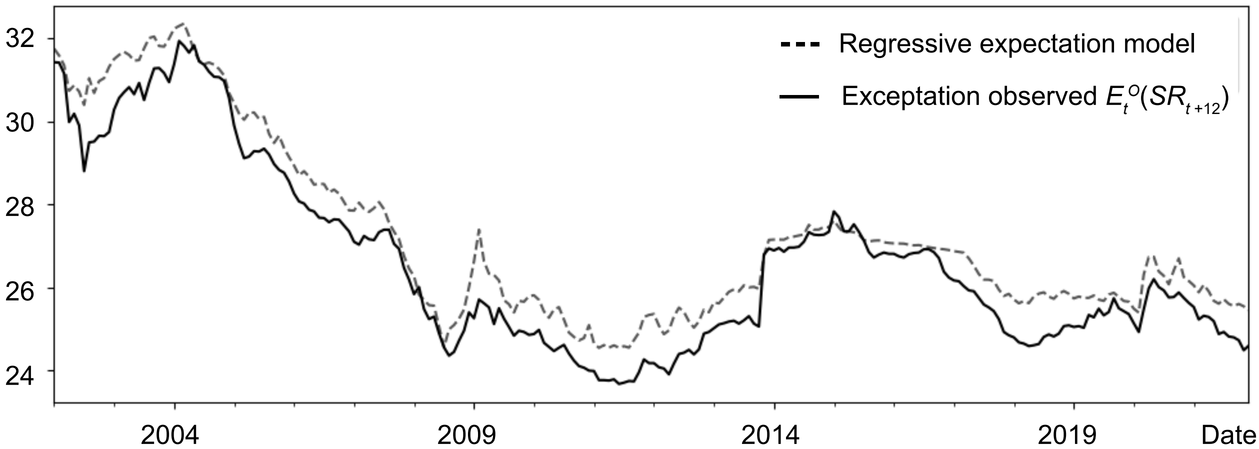
Notes: The figures display the expected levels indicated by the models, along with the expectations observed in the data $E_t^O(SR_{t+12})$.
Source: Author's own elaboration; based on CNB (2023a) and CNB (2023b)

Figure 6: Comparison of observed and theoretical data for static expectation model



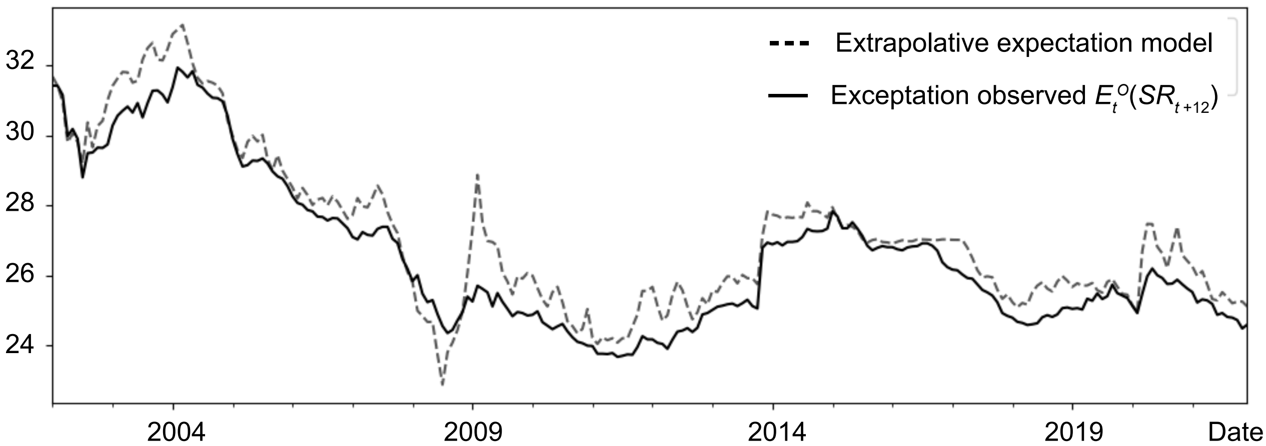
Notes: The figures display the expected levels indicated by the models, along with the expectations observed in the data $E_t^O(SR_{t+12})$.
Source: Author's own elaboration; based on CNB (2023a) and CNB (2023b)

Figure 7: Comparison of observed and theoretical data for regressive expectation model



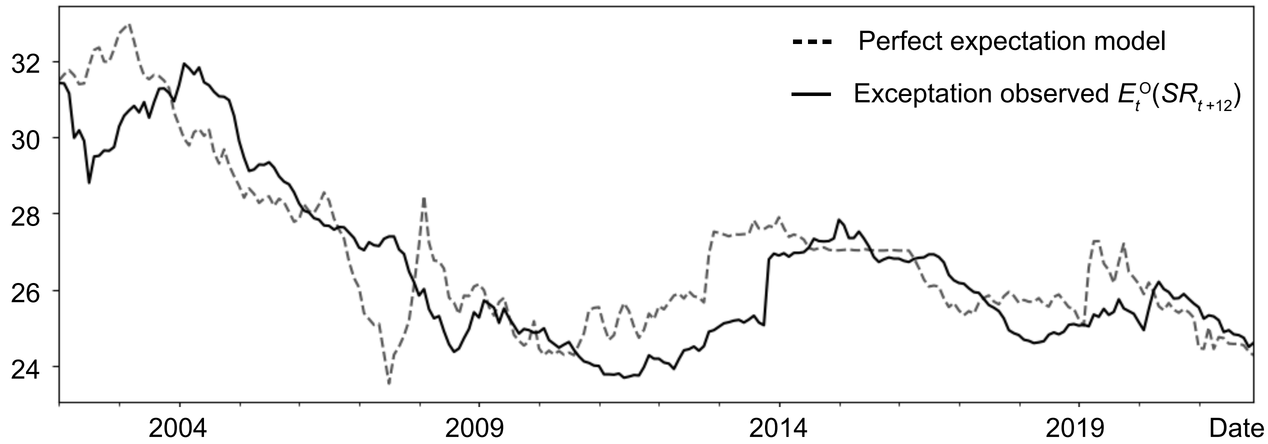
Notes: The figures display the expected levels indicated by the models, along with the expectations observed in the data $E_t^O(SR_{t+12})$.
Source: Author's own elaboration; based on CNB (2023a) and CNB (2023b)

Figure 8: Comparison of observed and theoretical data for extrapolative expectation model



Notes: The figures display the expected levels indicated by the models, along with the expectations observed in the data $E_t^O(SR_{t+12})$.
Source: Author's own elaboration; based on CNB (2023a) and CNB (2023b)

Figure 9: Comparison of observed and theoretical data for perfect expectation model



Notes: The figures display the expected levels indicated by the models, along with the expectations observed in the data (SR_{t+12}).
Source: Author's own elaboration; based on CNB (2023a) and CNB (2023b)