
Testing Real Interest Rate Parity for EU5 Countries: 200 Years of Data, Non-normality, Non-linearity and Breaks

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Abstract

Purpose: This paper aims to examine the real interest rate parity (RIP) theory for EU5 countries (France, Germany, Italy, Spain and the UK) versus the USA. **Design/methodology/approach:** Utilizing RALS-FADF and RALS-FKSS unit root tests, this study addresses non-normality, non-linearity and structural breaks in real interest rate differentials. **Findings:** The results confirm the RIP theory, indicating mean reversion of real interest rate differentials and highlighting impact of financial integration on monetary policy independence and arbitrage opportunities. The study notes that central banks' ability to influence domestic economies through interest rates is limited due to global financial interconnectedness. **Originality/value:** The paper offers a new test and bases its empirical setup on whether interest rate differentials are non-normally distributed. The test also considers real interest rate non-linearity and the non-normality in the analysis.

Keywords: Real interest rate parity, unit root, Fourier approximation, structural breaks, European countries, RALS terms, non-linearity

JEL Classification: C22, E43, F41

1. Introduction

The increasing integration of financial markets since the 1970s has significantly influenced the global economy. Easing restrictions on the flow of financial money has made this integration easier. The demise of the Bretton Woods system and subsequent adoption of free exchange played a role in the globalization of commodities and financial markets. Countries' decisions to liberalize and open up their domestic financial systems have fuelled this trend towards financial globalization. Overall, these modifications have improved financial market interconnectivity and capital mobility (Wu and Chen, 1998; Cuestas and Harrison, 2010; Jiang *et al.*, 2013).

Real interest rate parity (RIP) is a fundamental concept for understanding the equalization of real interest rates between countries, assuming perfect arbitrage and rational expectations. Cumby and Obstfeld's (1982) foundational study, as well as Lee and Tsong's (2012) subsequent analysis, highlighted the theoretical significance of RIP in the context of global financial integration. However, empirical study has yielded varying results. Earlier studies were usually suspicious of the empirical validity of RIP due to the simplicity of the testing methods used, such as classical regression analysis and standard unit root tests (Mishkin, 1984; Mark, 1985).

RIP has been re-examined with an emphasis on correcting past methodological flaws, spurred by recent developments in econometric approaches. To test the idea, this body of literature has started to investigate the implications of data distributional features, non-linearity and structural breaks. However, a thorough investigation employing sophisticated econometric techniques and long-span data is still lacking.

More sophisticated empirical approaches, including possible structural breaks and non-linearities, have been used to evaluate the RIP hypothesis. Changes in the foreign exchange risk premium, the global financial crisis and instabilities in the financial system, domestic policy changes and government interventions in the foreign exchange market and the Balassa-Samuelson effect, which explains the change in the portfolio allocation decisions of market participants due to real shocks and productivity changes, could all cause structural breaks in the time series properties of the real interest rate differential (Xie *et al.*, 2023).

The presence of transaction costs in arbitrage activities that cause real interest rates to drift away, asymmetric information on financial markets, the existence of non-tradable goods and services, and the size of real interest rate differentials are said to be the causes of potential non-linearities and play a significant role in explaining real interest rate differentials and in testing the RIP hypothesis (Çorakcı *et al.*, 2017, and citations therein).

Prior research has thoroughly explored the real interest rate concept and its consequences. However, a gap in the literature still remains. The RIP hypothesis has been supported or refuted

by numerous studies examining the interplay between interest rates and currency exchange rates. Although much has been learned, there are still questions that need to be answered.

This study aims to fill this gap by applying novel econometric methods to a long-span dataset from 1800 to 2018, comparing EU5 countries (France, Germany, Italy, Spain and the UK) with the USA. We utilize Fourier ADF and Fourier KSS unit root tests, which improve the power of traditional tests by accounting for potential smooth breaks and non-linear patterns in real interest rate differentials. EU countries provide an ideal laboratory since they have been relatively open for more years (Jenkins and Madzharova, 2008) and the European Union is a special case for financial integration with a single currency (Baharumshah *et al.*, 2005).

This study significantly enhances the existing literature through multiple contributions. Firstly, we build our empirical setup based on whether the interest rate differentials are non-normally distributed because the traditional stationarity and unit root tests have low power as the non-normality in the residuals is not considered. Secondly, in accordance with recent developments in earlier works, we paired the non-normality analysis with a test to determine whether real interest rate differentials exhibited a non-linear pattern. Thirdly, to capture the reaction of the interest rate differential series, structural breaks are not thought of as sharp drifts, but rather as smooth processes. So, we also consider the smooth breaks in the data generation process. More specifically, we estimate the Fourier ADF and Fourier KSS unit root test equations of Christopoulos and León-Ledesma (2010) using the residual augmented least squares method to improve the power of the tests. Finally, we test the RIP hypothesis using a long-span dataset which consists of 219 years of data.

The rest of this paper is organized as follows. The following section reviews the literature. The third section provides the theoretical framework of the RIP hypothesis. The fourth section introduces the empirical methodology. The fifth section discusses the data and provides the empirical results. The final section concludes the paper and summarizes the key findings.

2. Literature Review

Empirical plausibility of the RIP hypothesis has several implications. Firstly, the degree of international and interregional financial integration is measured using the RIP hypothesis (see Chinn and Frankel, 1995; Phylaktis, 1999; Xie *et al.*, 2023). Secondly, the existence of the RIP also hinders central banks' freedom to set interest rates and thus influences macroeconomic factors (Mark, 1985; Bagdatoglou and Kontonikas, 2011). Thirdly, policymakers' efforts to boost saving rates and spur capital formation via interest rate hikes may be futile, given that the empirical validity of the RIP predicts the convergence of domestic interest rates to global rates (Feldstein, 1982; Çorakcı, 2017). Fourthly, the RIP hypothesis elucidates international portfolio diversification and

hedging options (Balke and Wohar, 1998; Güney and Hasanov, 2014). Finally, the RIP hypothesis is a robust approximation for the determination of the exchange rate in monetary and portfolio balance models, as demonstrated by Dornbush (1976), Frenkel (1976) and Mussa (1976).

However, several arguments also contend that the RIP hypothesis may not be valid for a number of reasons. Firstly, due to financial investors' risk aversion, securities, even those with the same maturity and liquidity, may offer varying risk premia between countries. Secondly, interest rates may deviate from the global interest rates due to irrational behaviour of economic agents and the presence of non-tradable goods and services (for detailed discussions, see Fama and Farber, 1979; Hansen and Hodrick, 1980; Dornbusch, 1983; Alper *et al.*, 2009; Güney and Hasanov, 2014).

Examining and debating the viability of the RIP hypothesis is common because of its significant role in open economy macroeconomics and its ramifications for economic agents and policymakers. Since the RIP hypothesis assumes convergence of real returns across countries because of market participants' arbitrage operations, its veracity demonstrates that real interest rate differentials have mean-reverting stochastic processes (Çorakcı *et al.*, 2017).

Initial explorations of the RIP hypothesis were largely sceptical, citing frequent deviations from parity conditions observed in empirical data. Early analyses by Cumby and Obstfeld (1984), Mishkin (1984), Mark (1985) and Meese and Rogoff (1988) used regression analysis and traditional unit root tests, often concluding against the hypothesis due to their limited methodological approach.

To address this issue, further studies have treated the cointegration methodology (*e.g.*, Goodwin and Grennes, 1994; Moosa and Bhatti, 1996) but they could not provide a definitive solution. Their findings were also questioned because unit root tests on pooled cross-sectional datasets might significantly increase statistical power, while these traditional unit root tests may experience power loss (Wu and Chen, 1998; Holmes, 2002; Baharumshah *et al.*, 2005).

The narrative began to shift as methodological advances in econometrics allowed more sophisticated analyses. Studies began incorporating cointegration techniques and panel data approaches, which provided more robust evidence in support of RIP under certain conditions (Fountas and Wu, 1999; Wu and Fountas, 2000). For the economies of Europe and the G7 countries, Fountas and Wu (1999) and Wu and Fountas (2000) used panel cointegration tests that account for structural breaks in real interest rate differentials and arrived at conclusions that provide evidence in favour of the RIP.

However, subsequent studies in the literature have used novel methods, paid more attention to the possibility of structural breaks and non-linearities in real interest rate differentials, and provided more support favouring the RIP hypothesis. Therefore, the early findings on the rejection of the RIP hypothesis can be attributed to the poor performance of conventional unit root tests

because time series might exhibit structural changes, non-linearity and non-normality (Fu *et al.*, 2016).

Several studies have discovered evidence of the role of non-linearity in the mean-reverting process of real interest rate differentials. Several studies have been conducted, including Holmes and Maghrebi (2005) for nine OECD countries, Baharumshah *et al.* (2008) for ASEAN-5 countries, Baharumshah *et al.* (2009) for G7 and Asian countries, Cuestas and Harrison (2010) and Jiang *et al.* (2013) for Central and Eastern European countries, Fu *et al.* (2016) for Asian countries, Güney and Hasanov (2014) and Çorakcı *et al.* (2017) for G7 and post-Soviet countries, Nazlioglu *et al.* (2022) for 21 emerging economies and You *et al.* (2023) for the USA, the UK, Japan, Switzerland and the Eurozone. Furthermore, Liu and Lee (2022) studied the dynamic relationships between exchange rates and interest rates, with a special emphasis on China and the USA. Using a rolling-window technique for their empirical study, they investigated how variations in interest rates influence exchange rate volatility and vice versa, discovering that uncovered interest parity (UIP) holds in some subperiods but not over the full sample.

To capture structural breaks as a smooth process and improve the reliability of their tests for the quantity and form of structural breaks in real interest rate differential series, researchers in the second strand of the literature have used Fourier terms in their analyses. Su *et al.* (2012) used this technique for 12 Central and Eastern European countries, Chang and Su (2015) for G7 countries, Bahmani-Oskooee *et al.* (2019) for OECD and BRICS countries, and Xie *et al.* (2023) for Pacific Rim countries. These studies have used strong empirical methods to deal with structural fractures when investigating the validity of the RIP hypothesis. Furthermore, Vo and Vo (2023) discussed how, despite variations in currency prices and trade restrictions, arbitrage and resource allocation ultimately stabilize these distortions over time. They investigated several major problems in international economics that highlight the important relationship between exchange rates and price levels, strengthening the practical utility of purchasing power parity in modern economic analysis.

There is still an unavoidable dependence on the methodology, sample size and countries investigated by different articles when assessing the empirical support for the RIP hypothesis. Studies that account for non-normality, non-linearity and the existence of structural fractures in various forms are required to make a meaningful contribution to the field.

3. Theoretical Background

The RIP hypothesis assumes that domestic real interest rates do not deviate from the “global” real interest rate, as foreign arbitrage opportunities on the financial and goods markets prevent such deviations from occurring (Bagdatoglou and Kontonikas, 2011).

Two crucial ideas in international finance are uncovered interest parity (UIP) and purchasing power parity (PPP). The UIP theory asserts that the anticipated variation in the exchange rate is equal to the difference between domestic and international interest rates. PPP asserts that given sufficient time and free trade between countries, the prices of products and services in those countries will converge. To obtain RIP, a crucial metric for gauging international capital flow, we combine PPP, UIP and the Fisher equation (Arghyrou *et al.*, 2009; Shi *et al.*, 2012).

The UIP hypothesis states that, in the long run, there should be no arbitrage between local and foreign currency-denominated assets since interest rate spreads within countries should converge. According to the uncovered interest parity on FX markets, the expected change in the exchange rate is equal to the difference between the domestic and foreign interest rates (Jiang *et al.*, 2013). In conjunction with PPP, it denotes convergence of real interest rates (Wu and Chen, 1998). Understanding and managing global economic dynamics requires a firm grasp of these ideas, which sheds light on the interplay between interest rates, currency rates and international capital flows.

The real interest rate parity hypothesis combines the Fisher equation, the UIP and international PPP conditions. The theoretical equations for RIP can be expressed as follows (Cumby and Obstfeld, 1982).

According to the UIP theory, when economic agents form rational expectations about exchange rates, the interest rate differential between two countries should be equal to the anticipated change in the exchange rate between their respective currencies.

$$i_t = i_t^* + \frac{E_t^e - E_t}{E_t} + e_t \quad (1)$$

where i_t , i_t^* , E_t and E_t^e demonstrate the domestic interest rate, foreign interest rate, exchange rate (current) and the expected future exchange rate, respectively and e_t is the error term distributed as $e_t \sim iid(0, \sigma^2)$.

The second term in Equation (1) is the expected rate of depreciation, d_t^e :

$$i_t = i_t^* + d_t^e \quad (2)$$

However, PPP contends that the exchange rate should be adjusted to bring the costs of comparable items in the two countries into line.

$$E_t = \frac{P_t}{P_t^*} \quad (3)$$

where P_t is the domestic price level and P_t^* is the foreign price level.

By combining these equations, we obtain:

$$\pi_t^e - \pi_t^{*e} = d_t^e \quad (4)$$

where π_t^e is the expected rate of change of domestic prices, namely the domestic inflation rate, and π_t^{*e} is the expected rate of change of foreign prices, namely the foreign inflation rate.

The Fisher equation states that the nominal interest rate at the time t can be written as:

$$i_t = r_t + \pi_t^e \text{ or } r_t = i_t - \pi_t^e \quad (5)$$

where r_t , i_t and π_t^e denote the real interest rate, the nominal interest rate and the expected inflation rate, respectively.

When we combine the UIP condition, the PPP condition and the Fisher equations

$$i_t - i_t^* = r_t - r_t^* = rid_t = e_t \quad (6)$$

Equation (6) implies that when the UIP, PPP and Fisher equations hold so that the RIP hypothesis is valid, the differentials of real interest rate differentials (rid_t) follows a stationary process with a mean of zero.

4. Econometric Methodology

4.1 More powerful Fourier ADF and Fourier KSS unit root tests

Non-normal error terms do not affect the limiting distribution of traditional unit root tests; hence, these tests ignore information in non-normal error terms¹. Im and Schmidt (2008) proposed the residual augmented least squares (RALS) method to use the information by considering the higher moments of the residuals in the testing regression. The RALS is fairly simple to apply because the testing procedure is carried out within a linear framework based on the least-squares estimate methodology. Here, we enhance Christopoulos and León-Ledesma's (2010) two-step method for Fourier unit root testing by adapting the RALS method and improving the power of the tests in the case of non-normal error distributions.

1 Paparoditis and Politis (2003) argued that in the case of a strong negative correlation, there are size distortion problems of the unit root tests even when using block bootstraps. Besides, bootstraps are computer-intensive simulation techniques (see Barreto and Howland, 2006); hence, to replicate a study which used bootstrap-based critical values will not be eligible for researchers with no software knowledge. To allow further studies to employ our suggested tests and also to replicate our findings, we computed critical values via Monte Carlo simulations.

In the first step, the following test regression model is considered.

$$y_t = \alpha(t) + u_t \quad (7)$$

where $\alpha(t)$ is a time-varying deterministic term which can be expanded as follows and used to approximate the unknown number of breaks of the unknown form:

$$\alpha(t) = \alpha_1 + \sum_{j=1}^n \alpha_2^j \sin(2\pi kt/T) + \sum_{j=1}^n \alpha_3^j \cos(2\pi kt/T) \quad (8)$$

where n and k denote the number of frequencies and a particular frequency, respectively; α_2 and α_3 quantify the amplitude and displacement of the frequency component; t and T represent the trend and sample size, respectively. This Fourier function is used to capture the behaviour of unknown breaks of unknown forms (see, *e.g.*, Gallant, 1984; Davies, 1987; Becker *et al.*, 2004, 2006; Enders and Lee, 2012a, 2012b). When $n = T/2$, the approximation of $\alpha(t)$ is very good. However, following Ludlow and Enders (2000), we allow a single frequency, because it can approximate the Fourier expansion. Therefore, we can rewrite Equation (8) as:

$$\alpha(t) = \alpha_1 + \alpha_2 \sin(2\pi kt/T) + \alpha_3 \cos(2\pi kt/T) \quad (9)$$

Substituting the Fourier function from (9) into Equation (7) yields the following model:

$$y_t = \alpha_1 + \alpha_2 \sin(2\pi kt/T) + \alpha_3 \cos(2\pi kt/T) + u_t \quad (10)$$

Ludlow and Enders (2000) showed that Equation (10) can capture various types of non-linearities; Becker *et al.* (2004) suggested that Equation (10) has the correct empirical size and excellent power to detect multiple structural changes of unknown form. As k is not known a priori, we estimate Equation (10) for each integer value of k ranging from 1 to 5. We then choose the value of k that minimizes the sum of squared residuals (*SSR*).

After determining the appropriate frequency (k^*), we obtain the OLS residuals of Equation (11).

$$\hat{u}_t = y_t - [\hat{\alpha}_1 + \hat{\alpha}_2 \sin(2\pi k^* t/T) + \hat{\alpha}_3 \cos(2\pi k^* t/T)] \quad (11)$$

In the second stage, we estimate the ADF and KSS test regressions to test for a unit root of the residuals. The regression equations are as follows:

$$\Delta u_t = \beta_1 u_{t-1} + \sum_{i=1}^p \delta_i \Delta u_{t-i} + e_t \quad (12)$$

$$\Delta u_t = \varphi_1 u_{t-1}^3 + \sum_{i=1}^p \zeta_i \Delta u_{t-i} + e_t \quad (13)$$

where Δ indicates the difference operator and ρ denotes the optimal lag length. Equations (12) and (13) are the test equations of Fourier ADF and Fourier KSS unit root tests suggested by Christopoulos and Leon-Ledesma (2010), which allow for multiple structural changes. However, in this study, we will take a step forward by proposing a new test in which we estimate Equations (12) and (13) using the RALS methodology. In the next step, the second and third moments of errors of Equations (12) and (13) are obtained and included in the same equation as follows:

$$\Delta u_t = \delta_1 u_{t-1} + \sum_{i=1}^p \phi_i \Delta u_{t-i} + \hat{w}_t' \gamma + v_t \quad (14)$$

$$\Delta u_t = \varphi_1 u_{t-1}^3 + \sum_{i=1}^p \zeta_i \Delta u_{t-i} + \hat{w}_t' \gamma + \omega_t \quad (15)$$

where $\hat{w}_t = [\hat{e}_t^2 - m_2, \hat{e}_t^3 - m_3 - 3m_2\hat{e}_t]'$. $\hat{e}$ is the residuals of Equation (12) or (13), m_2 and m_3 denote the mean of the squared residuals and the mean of the cube of the residuals, respectively. The null hypothesis of the unit root of $H_0: \delta_1 = 0$, against the alternative of $H_1: \delta_2 < 0$ can be tested t -statistic $\tau = \hat{\delta}_1 / se(\hat{\delta}_1)$, where se denotes the standard error for $\hat{\delta}_1$ in Equation (14) and $H_0: \varphi_1 = 0$ can be tested against the alternative of $H_0: \varphi_1 < 0$ using the test statistic $\tau = \hat{\varphi}_1 / se(\hat{\varphi}_1)$, where se represents the standard error for $\hat{\varphi}_1$ in Equation (15). The initial component of $\hat{w}_t$ is linked to the homoscedasticity assumption and enhances the estimator efficiency of α when the errors are asymmetric. The second component of $\hat{w}_t$ improves the efficiency as long as $E(e_t^4) \neq 3\sigma^4$. Im *et al.* (2014) suggested that the size-adjusted power of the RALS unit root test is similar to that of the traditional unit root test when the error is normally distributed; however, it has more power when the error exhibits a non-normal distribution. We refer to this test based on Equation (14) as the RALS-FADF test and the test based on Equation (15) as the RALS-FKSS test.

We obtain the critical values for the RALS-FADF and RALS-FKSS unit root tests for different values of k ranging from 1 to 5, with sample sizes of 100, 250 and 500 and $\hat{\rho}^2$ values for 0.1 to 1 using 100,000 Monte Carlo simulations. The critical values are presented in Appendix I.

To use the critical values, we must estimate the correlation coefficient between the residuals of the Fourier-based test and the RALS-based test as:

$$\hat{\rho}^2 = \hat{\sigma}_*^2 / \hat{\sigma}^2 \quad (16)$$

where $\hat{\sigma}^2$ indicates the estimate of the error variance in the FADF or FKSS test regressions and $\hat{\sigma}_*^2$ denotes the estimate of the error variance in the RALS-FADF or RALS-FKSS test regressions.

To test the null hypothesis of the non-significance of the trigonometric terms $H_0: \alpha_2 = \alpha_3 = 0$ in Equation (10), we employ the F -test, which is based on the estimation of Equation (10) via ordinary least squares, and compute the following test statistics:

$$F_{trig}^{OLS} = \frac{(SSR_r - SSR^*)/2}{SSR^*/(T - q - 1)} \quad (17)$$

where SSR_r is the SSR of Equation (10) without trigonometric terms, while SSR^* is the SSR of Equation (10), and q denotes the number of parameters in Equation (10). The F -test should be used in the case of stationary data; that is, after determining that the series is stationary, the significance of trigonometric terms should be tested. The critical values for F_{trig}^{OLS} were tabulated by Becker *et al.* (2006). If the trigonometric terms are found to be non-significant, then we can use the RALS-ADF or RALS-KSS unit root tests that correspond to the RALS-FADF and RALS-FKSS unit root tests without a Fourier function.

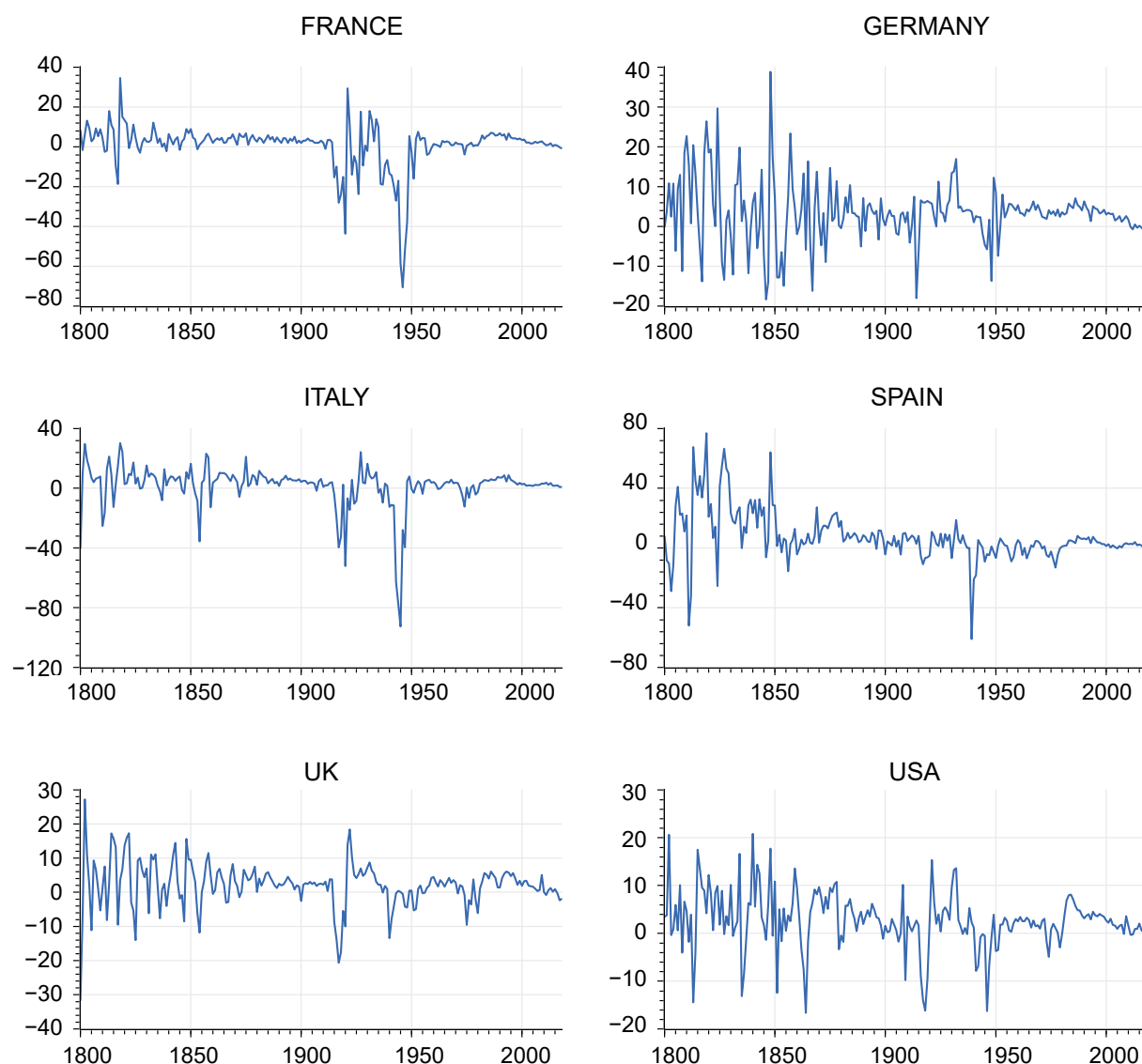
The RALS-FADF and RALS-FKSS unit root tests allow for multiple structural breaks and have more power than the FADF and FKSS unit root tests when the errors are non-normal².

5. Data and Empirical Results

We use Schmelzing's (2019) data on real interest rates for the EU5 countries (France, Germany, Italy, Spain and the UK) for a long span of data from 1800 to 2018 to test the real interest parity hypothesis. We then compute the real interest differentials using the real interest rates of the USA as the base country. Figure 1 illustrates the real interest rates of the EU5 countries and the USA.

The trajectory of real interest rates in France exhibits marked volatility throughout the observed timeframe. Early surges coincide with periods of geopolitical instability, such as the Napoleonic Wars. The 20th century witnessed fluctuations linked to global conflicts such as the World Wars and their subsequent economic ramifications. Similarly, Germany's real interest rate demonstrates substantial volatility, particularly during pivotal historical junctures including the unification period, the World Wars and the post-war reconstruction era. The reunification of East and West Germany towards the end of the 20th century is evidenced by a distinct decline in real interest rates followed by a gradual recovery. Italy's real interest rate follows a pattern of fluctuations, characterized by notable dips during both World Wars and the ensuing post-war periods. The latter half of the 20th century reveals a phase of relative stability, eventually transitioning into a downward trend towards the end of the observed period. Spain's real interest rate displays significant volatility throughout the 19th and early 20th centuries, reflecting epochs of political turbulence and internal strife, notably the Spanish Civil War.

2 To preserve space, we do not present the small sample properties. The findings show that the proposed tests are powerful especially in the case of non-normal errors in the different data generation processes.

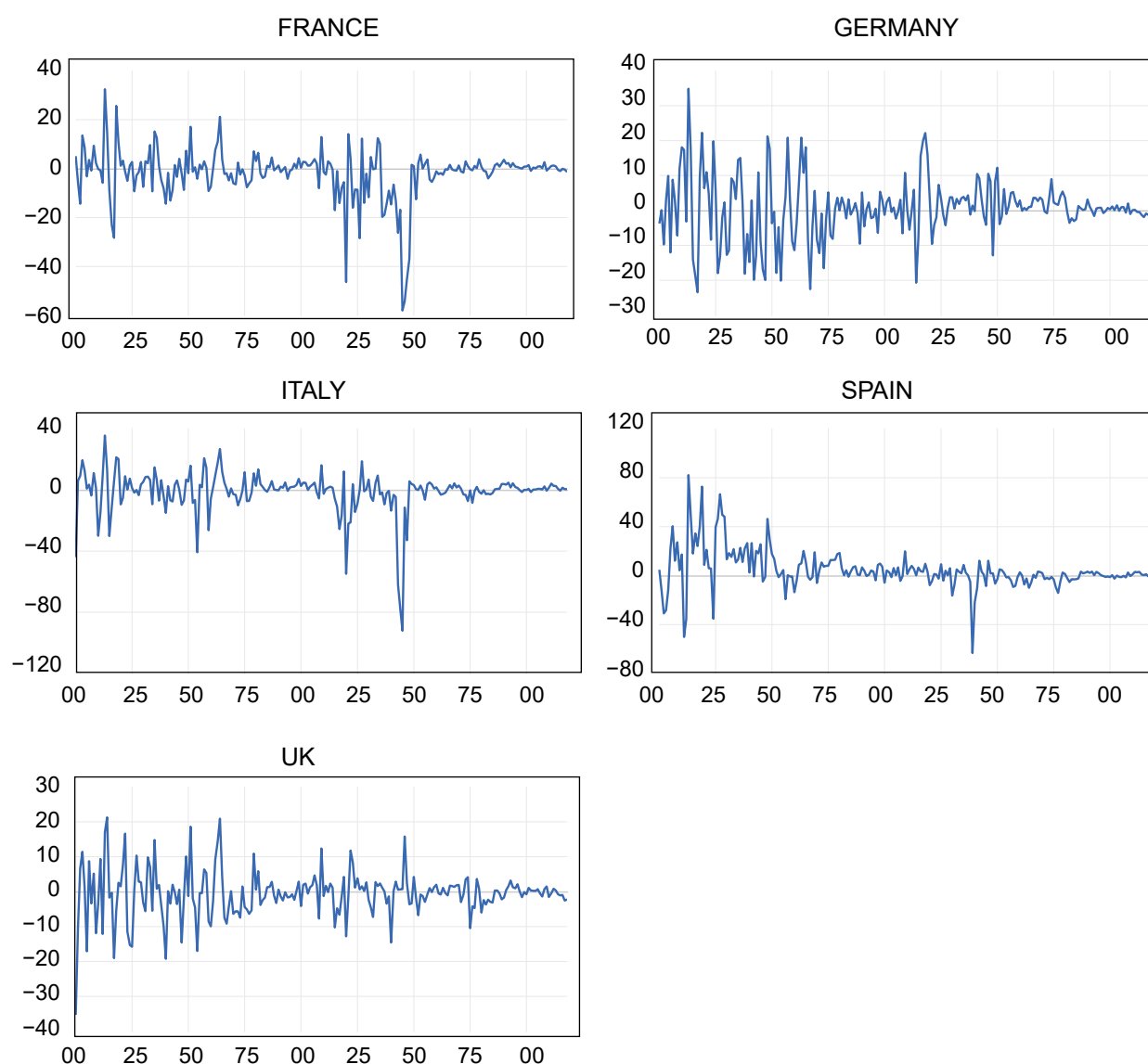
Figure 1: Time path of real interest rates

Source: Schmelzing's (2019) data on real interest rates for the EU5

The latter part of the 20th century demonstrates greater stability, punctuated by fluctuations coinciding with the European integration process. The USA, while not immune to fluctuations, particularly during the Civil War, the World Wars and periods of economic contraction and expansion, exhibits a greater degree of stability in its real interest rate compared to its European counterparts. Across all the examined countries, the real interest rates exhibit notable volatility, particularly during periods of significant geopolitical turmoil and economic crises. However, the magnitude of this volatility varies across countries. France, Germany and Spain experienced more pronounced fluctuations, whereas the UK and the USA demonstrate comparatively greater stability.

We also provide a plot of interest rate differentials of EU countries with respect to the USA in Figure 2.

Figure 2: Time path of real interest rate differentials with respect to USA



Note: Real interest rate differentials are on the X-axis.

Source: Authors' own classification

A preliminary visual analysis of the interest rate differentials between the five European countries and the USA suggests the presence of characteristics consistent with stationarity. Specifically, the observed data exhibit a tendency towards mean reversion, as evidenced by the general inclination of the interest rate differentials to fluctuate around a constant mean, effectively

gravitating towards the zero line over time. Furthermore, the series demonstrates relative constancy in variance, with fluctuations exhibiting a relatively consistent spread throughout the observed timeframe. The absence of recognizable long-term trends further supports the notion of stationarity. Table 1 presents the descriptive statistics of interest.

Table 1: Descriptive statistics

	Mean	Median	Maximum	Minimum	Std. dev.	Skewness	Kurtosis	Jarque-Bera	Probability
France	0.787	2.693	34.485	-70.673	11.341	-2.855	16.187	1884.292	0
Germany	3.411	3.349	38.839	-18.361	7.799	0.446	5.907	84.343	0
Italy	1.531	3.741	30.182	-92.732	14.130	-3.275	18.624	2618.980	0
Spain	7.417	4.351	76.842	-61.020	16.758	0.841	7.520	212.202	0
UK	2.189	2.470	27.225	-31.797	6.504	-0.731	7.714	222.233	0
USA	2.738	2.664	20.755	-16.651	5.835	-0.334	5.284	51.665	0

Source: Schmelzing's (2019) data on real interest rates for the EU5

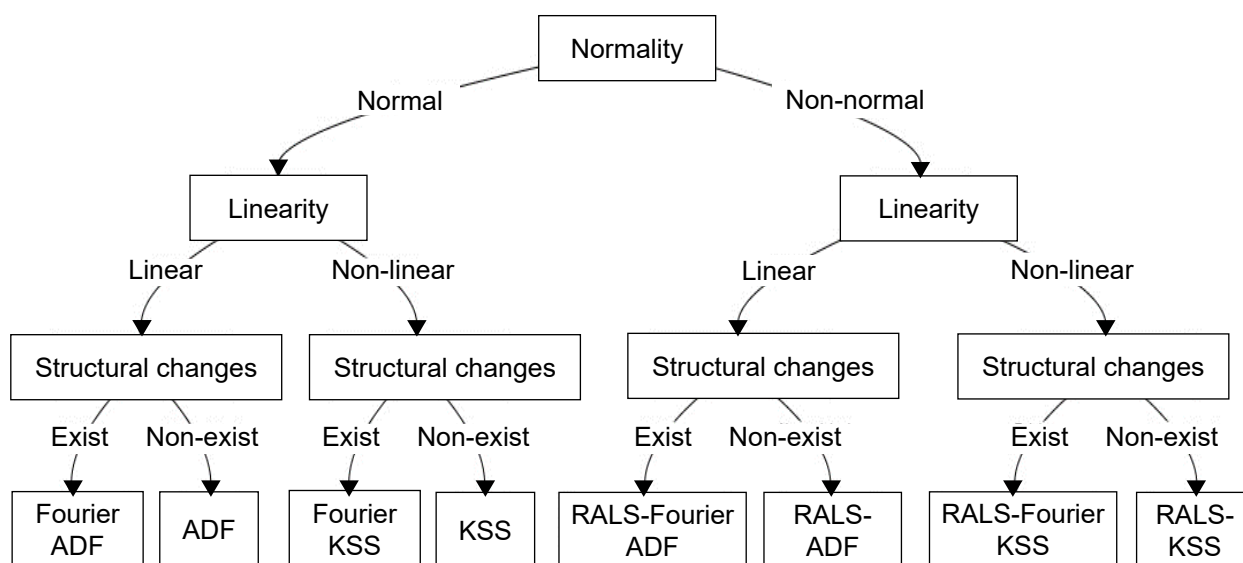
Table 1 shows that the positive mean values for all the countries in the analysis suggest a general trend of positive real interest rates over the observed period. This phenomenon reflects the fundamental concept of the time value of money, which posits that investors expect compensation for lending their capital over a given time horizon. The variations in mean values across the countries highlight the inherent differences in their respective economic conditions, inflation rates and monetary policies, all of which play a crucial role in determining the prevailing interest rates. An examination of the standard deviations reveals notable variations in the volatility of interest rates among the countries studied. France, Italy and Spain exhibit higher standard deviations, suggesting that their interest rates are subject to greater fluctuations. These fluctuations may be attributed to various factors, such as economic instability, political events or external shocks, which can introduce uncertainty and volatility into the interest rate environment. Conversely, Germany, the UK and the USA demonstrate lower standard deviations, indicating the presence of more stable interest rate environments. This stability could be a reflection of stronger economic fundamentals and consistent monetary policies in these countries, which contribute to a more predictable interest rate landscape.

The positive skewness coefficients observed for Germany and Spain are particularly noteworthy, as they indicate a tendency for the interest rate distributions of these countries to have more frequent occurrences of lower rates, while also exhibiting the presence of some extreme high values. This phenomenon could be attributed to periods of economic stability characterized by low-interest rates, punctuated by episodes of economic stress or policy interventions that result in temporary spikes in interest rates. Such a pattern suggests that while these countries generally maintain lower interest rates, they are not immune to occasional bouts of economic turbulence that can lead to significant deviations from the norm. Furthermore, the kurtosis values exceeding 3 for all the countries in the study signify the presence of leptokurtic distributions.

These distributions are characterized by heavier tails compared to a normal distribution, implying a higher probability of observing extreme interest rate values (both high and low) than what would be expected under the assumption of normality. The occurrence of such extreme values could be associated with significant economic events, crises or policy shifts that have a profound impact on the interest rate environment. These events may introduce substantial volatility and lead to interest rates that deviate significantly from the mean, thereby contributing to the leptokurtic nature of the distributions. The Jarque–Bera test results, with p -values indicating statistical significance, provide conclusive evidence that the real interest rate series for all the countries deviate from a normal distribution.

The configuration of our empirical analysis to test the stochastic properties of real interest rate differentials (RIDs) is shown in Figure 3.

Figure 3: Testing strategy



Source: Authors' own classification

The first empirical step in our analysis is to test whether the RID series is normally distributed. We test the normality of the series using the Shapiro–Wilk test, which is declared one of the most powerful normality tests in the literature (see Mendes and Pala, 2003; Yap and Sim, 2011; Razali and Wah, 2011)³. The existence of a non-normal distribution will direct us to incorporate the RALS methodology into our empirical procedure. The Shapiro–Wilk normality test results are presented in Table 2.

Table 2: Shapiro–Wilk normality test

Countries	Test stat.	<i>p</i> -values
France	0.790*	0
Germany	0.949*	0
Italy	0.716*	0
Spain	0.834*	0
UK	0.912*	0

Note: * represents significance at the 1% level.

Source: Author's own calculations with gathered data

The null hypothesis of normality is tested using the Shapiro–Wilk test. We can conclude that all the series are not normally distributed because the *p*-values of the test statistics are smaller than the commonly used significance levels. As a result, the Jarque–Bera test statistics in Table 2 are credible, as shown by the results of the Shapiro–Wilk normality test. Therefore, we can deduce that testing the stationarity of the RID series requires the use of RALS terms for a more robust analysis.

In the second stage of our empirical setup, we examine whether the adjustment of the RIDs to the equilibrium level is non-linear by testing the linearity of the series using the test of Harvey *et al.* (2008). The robustness of the test developed by Harvey *et al.* (2008) holds true for both stationary and non-stationary series. Table 3 displays the results of the test.

³ See Guner *et al.* (2009) for a summary of the Shapiro–Wilk test.

Table 3: Harvey *et al.* (2008) linearity test results

Country	Test stat.	<i>p</i> -values
France	4.681***	0.096
Germany	2.928	0.231
Italy	8.312**	0.016
Spain	3.022	0.221
UK	1.254	0.534

Note: *** and ** represent significance at the 10% and 5% levels, respectively.

Source: Author's calculations with gathered data

The null hypothesis of the test of Harvey *et al.* (2008) indicates the linearity of the series. As can be readily seen in Table 3, we reject the linearity in the cases of France and Italy and conclude that the RID series exhibit non-linearities. Thus, we should test the stationarity of RIDs in the cases of France and Italy, considering the non-linear data generation process. However, for the cases of Germany, Spain and the UK, we need to apply linear adjustment processes.

Table 4: RALS-FADF and RALS-FKSS test results

	FADF-RALS	FKSS-RALS	Frequency	Opt. lag	Rho	<i>F</i> -test stat
France	–	–4.129 *	1	0	0.756	8.840*
Germany	–7.610 *	–	1	1	0.864	2.265
Italy	–	–8.287 *	1	0	0.799	5.292*
Spain	–5.047 *	–	1	0	0.725	20.928*
UK	–6.517 *	–	5	3	0.841	0.842

Note: * represents significance at the 1% level.

Source: Author's own calculations with gathered data

Having the stochastic properties of RIDs related to non-normality and non-linearity, we can categorize the RID series into groups according to our empirical procedure (Figure 2). For the cases of France and Italy, we handle non-linear stationarity tests, allowing structural breaks as smooth and gradual processes (FKSS) and incorporated with RALS terms, namely the RALS-FKSS test. On the other hand, for Germany, Spain and the UK, we utilize linear stationarity tests

allowing structural breaks as smooth and gradual processes (FADF) and incorporated with RALS terms, namely the RALS-FADF test. The results of the RALS-FADF and RALS-FKSS tests are listed in Table 4.

Because the critical F -values can be used in the case of stationary variables, we first focus on the RALS-FADF and RALS-FKSS test statistics and then test the significance of the trigonometric terms if the series is stationary.

The test findings for FADF-RALS in Germany, Spain and the UK are presented in the first column of Table 4. We reject the null hypothesis that the series have unit roots for Germany, Spain and the UK, as shown by the relevant test statistics. Similarly, the null hypothesis of the unit root is rejected for the real interest rate differential between France and Italy, implying that it is stable around a non-linear trend and supporting the RIP hypothesis.

We then discuss the significance of trigonometric terms after establishing via an analysis of series stationarity that all the real interest rate differentials exhibit a mean-reverting pattern. The final column of Table 4 shows that all the F-test statistics for France, Italy and Spain reject the null hypothesis, demonstrating the significance of the trigonometric terms for these countries. In other words, the key trigonometric terms reveal a smooth and gradual trend characterizing the structural breaks in real interest rate differentials in France, Italy and Spain.

However, for the series of Germany and the UK, trigonometric terms are insignificant, as proved by not rejecting the null hypothesis and demonstrated by the related F-statistics. Therefore, we need to employ the RALS-ADF test, as suggested by Im *et al.* (2014), because the trigonometric terms are found to be insignificant. Table 5 presents the empirical findings of the RALS-ADF test.

Table 5: RALS-ADF test results

	ADF-RALS	Opt. lag	ρ
France	–	–	–
Germany	–13.101*	1	0.883
Italy	–	–	–
Spain	–	–	–
UK	–12.171*	3	0.822

Note: * represents significance at the 1% level.

Source: Author's own calculations with gathered data

The test statistics for real interest rate differentials provide support for the rejection of the null hypothesis of the unit root in series, as shown in Table 5. As a result, we conclude that although the RID series in Germany and Spain do not offer smooth and progressive structural discontinuities, they display a linear mean-reverting behaviour. As a result, in the examples of Germany and the UK, the RALS-ADF test results offer evidence consistent with the viability of the RIP hypothesis.

After conducting these unit root tests, we found that the RIP hypothesis is supported by the data. Several major conclusions can be drawn from the data obtained in this study. The study found that the likelihood of rejecting the unit root null hypothesis increased when non-normality, potential non-linearities and gradual structural breaks were included in the data collection process, as predicted by the RIP. It appears that real interest rate differentials may not adjust to the equilibrium level in a linear fashion for most of the series analysed, as seen by the occurrence of non-linearities in France and Italy. If the adjustment process is non-linear, this discovery has important implications on how foreign shocks affect these economies.

Our study shows that real interest rate differentials between the EU5 and the USA revert to the mean, implying a long-run equilibrium consistent with the RIP principles. This conclusion is consistent with previous research such as Xie *et al.* (2023), who discovered comparable patterns in various international contexts, confirming the robustness of our findings.

The empirical support for the RIP hypothesis points to strong financial integration and capital mobility between the EU5 and the USA. Because of the convergence of real interest rates across these countries, central banks in these countries may find it difficult to independently influence macroeconomic variables through national monetary policies. Such findings support those of Bahmani-Oskooee *et al.* (2019), who identified monetary policy restrictions in economically linked regions.

Our findings imply that mean-reverting real interest rate differentials make efficient cross-border hedging, arbitrage and portfolio diversification methods possible, which can be useful for investors on global financial markets. This scenario is consistent with the conclusions of studies such as Güney and Hasanov (2014), who emphasized the advantages of financial market integration for investment strategies. However, national policies aiming at influencing savings rates and capital formation through interest rate adjustments may be ineffective since they are frequently negated by market participants' arbitrage operations.

Both uncovered interest parity (UIP) and relative purchasing power parity (PPP) were shown to be valid in this study, corroborating prior research findings in similar economic environments. Previous research by Moosa and Bhatti (1996) and Edison and Pauls (1993) revealed various degrees of support for these parities, depending on the economic situation and time periods

explored. Our findings contribute to this body of literature by verifying these disparities over a long time period using rigorous analytical approaches.

We also conducted the analysis using Germany as the base country for the other EU4 countries (France, Italy, Spain and the UK), covering the period from 1800 to 2018⁴. The results are presented in Appendix II. Similar to the analysis with the USA as the base country, the Shapiro–Wilk tests in Table II. A confirm non-normal distributions for all the RID series. Furthermore, Table II. B reveals non-linearity in the RIDs for Italy and Spain, suggesting complex adjustment processes towards equilibrium. The RALS-FADF and RALS-FKSS tests (Table II. C) provide evidence of stationarity for the RIDs of France and Italy, supporting the RIP hypothesis. However, the RIDs of Spain and the UK are found to be non-stationary (Table II. D), indicating a lack of mean reversion and contradicting the RIP hypothesis.

Interestingly, using Germany as the base country yields mixed results compared to the analysis with the USA as the base country. While the RIP holds for France and Italy in both scenarios, it is rejected for Spain and the UK when using Germany as the base. The confirmation of RIP for France and Italy against Germany aligns with the expectation of stronger product and financial market integration within the Eurozone. The shared currency likely contributes to closer economic ties and more pronounced convergence of real interest rates. The rejection of RIP for Spain and the UK, despite using a European base country, highlights the heterogeneity in economic integration within the EU. Factors such as pre-euro legacy effects, differing economic structures and specific policy choices could explain this divergence. The mixed support for RIP within the EU underscores the need for policymakers to consider the varying degrees of integration when formulating monetary and fiscal policies. A “one-size-fits-all” approach may not be effective. The results when using Germany as the base country provide a nuanced perspective on the RIP

4 We re-conducted the analysis using data from 1950 to 2018 to assess the robustness of our findings using a more recent data sample. The results are presented in Appendix III. The RIDs of Germany, Italy, Spain and the UK are stationary, confirming the RIP hypothesis. Notably, France, which exhibited mean reversion in the full sample, now shows non-stationarity. The confirmation of RIP for Germany, Italy, Spain and the UK in the 1950–2018 sample (using the USA as a base) suggests that post-war economic integration and globalization have played a significant role in promoting real interest rate convergence. The non-stationarity of France’s RIDs highlights the persistence of country-specific factors and potential structural differences that may hinder complete convergence. The results of the 1950–2018 sample offer a partial confirmation of the initial findings. The RIP holds more robustly for some countries (Germany, Italy, Spain and the UK) in the more recent period, possibly due to the intensification of global financial integration. However, the persistent non-stationarity for France suggests that the RIP hypothesis may not be universally applicable across all EU countries and time periods. This robustness check using a shorter data span adds valuable insights to the study. While the findings partially support the original conclusions, they also highlight the importance of considering the evolving nature of financial markets and the potential influence of specific historical events (such as the introduction of the euro) on real interest rate dynamics.

hypothesis within the EU. While the original results when using the USA as the base country pointed towards a strong overall confirmation of RIP, the additional analysis reveals a more complex picture. The findings suggest that the RIP hypothesis holds more robustly within the Eurozone, highlighting the role of the shared currency in fostering integration. However, the rejection of RIP for some EU countries against Germany points to the limitations of relying solely on broader EU affiliation as a driver of convergence in real interest rates.

6. Conclusion

This study tested the real interest rate parity (RIP) hypothesis for EU5 countries relative to the USA, covering a historical period from 1800 to 2018. Employing advanced econometric techniques such as the Fourier ADF and Fourier KSS unit root tests, the analysis revealed that real interest rate differentials among these countries exhibit mean-reverting behaviour, indicative of a long-term equilibrium in most cases, consistent with the RIP hypothesis.

By considering data complexity such as non-normality, non-linearity and structural breaks, the methodological rigour of this study improves the reliability of the results. The study tried to achieve a higher level of understanding of financial integration and interest rate dynamics by surpassing the constraints of conventional unit root testing through the incorporation of these advanced tests.

Our findings support the empirical validity of the RIP hypothesis, demonstrating considerable financial integration and capital mobility between the EU5 and the USA. However, it is critical to understand that these findings represent long-term trends affected by a wide range of historical and policy issues.

The results are more nuanced when the analysis is extended to include the other EU4 countries (France, Italy, Spain and the UK) over the same time span. While the RIP hypothesis applies to France and Italy in both cases, it is rejected for Spain and the UK when Germany is the base country.

For France and Italy, the RIP hypothesis is verified when Germany is employed as the base country, demonstrating strong financial integration within the Eurozone. The idea does not hold for Spain or the UK. Despite being a Eurozone member, Spain reflects the heterogeneity of economic integration inside the EU. For the UK, which has not joined the euro, the lack of RIP validity may reflect its assessment of the Eurozone's optimum currency area characteristics.

Furthermore, the findings suggest that in highly linked financial systems, individual central banks' ability to affect macroeconomic outcomes through interest rate changes may be considerably limited. This finding is consistent with the concept of rising monetary policy interdependence,

yet it does not completely rule out national monetary policy actions, but rather underscores the restrictions faced by global financial market integration.

The observed mean reversion in real interest rate differentials makes cross-border hedging, arbitrage and portfolio diversification techniques more feasible, which can benefit global financial market participants. However, the efficiency of these techniques should be contextualized within the larger market and geopolitical factors.

The analysis implies that policymakers' efforts to influence domestic economic conditions through interest rate adjustments may be less effective than expected in the environment of widespread financial integration. This insight does not reduce the importance of such metrics; rather, it places them within the larger context of global economic interdependence.

Overall, this study supports the RIP hypothesis as a useful tool for understanding and regulating global economic processes. It emphasizes the need for using long-term data and advanced econometric approaches to capture the intricate relationships between interest rates, currency rates and international capital flows. The findings advance our understanding of financial market integration and its implications for monetary policy and investment strategy.

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Appendixes

Appendix I

Table A: Asymptotic critical values for the RALS-KSS unit root test

1. For demeaned series

ρ^2	10%	5%	1%
0.1	-2.817	-2.159	-1.791
0.2	-3.016	-2.350	-1.998
0.3	-3.096	-2.465	-2.129
0.4	-3.193	-2.581	-2.254
0.5	-3.275	-2.661	-2.345
0.6	-3.357	-2.739	-2.427
0.7	-3.369	-2.802	-2.497
0.8	-3.420	-2.848	-2.554
0.9	-3.437	-2.902	-2.617
1.0	-3.478	-2.940	-2.658

Note: Critical values are obtained for sample size 1 000 running 100 000 simulations.

2. For detrended series

ρ^2	10%	5%	1%
0.1	-2.980	-2.311	-1.956
0.2	-3.222	-2.563	-2.213
0.3	-3.375	-2.721	-2.395
0.4	-3.490	-2.877	-2.552
0.5	-3.587	-3.001	-2.688
0.6	-3.710	-3.113	-2.799
0.7	-3.759	-3.191	-2.891
0.8	-3.837	-3.280	-2.989
0.9	-3.887	-3.332	-3.052
1.0	-3.943	-3.406	-3.126

Note: Critical values are obtained for sample size 1 000 running 100 000 simulations.

Table B: Critical values for the RALS-FADF unit root test**1. For demeaned series**

K = 1				T = 100				T = 250				T = 500			
ρ^2	10%	5%	1%	ρ^2	10%	5%	1%	ρ^2	10%	5%	1%	ρ^2	10%	5%	1%
0.1	-3.052	-2.391	-2.027	0.1	-3.050	-2.373	-2.012	0.1	-3.04	-2.371	-2.007	0.1	-3.04	-2.371	-2.007
0.2	-3.330	-2.682	-2.330	0.2	-3.333	-2.661	-2.308	0.2	-3.319	-2.663	-2.302	0.2	-3.319	-2.663	-2.302
0.3	-3.547	-2.903	-2.561	0.3	-3.539	-2.879	-2.533	0.3	-3.532	-2.875	-2.527	0.3	-3.532	-2.875	-2.527
0.4	-3.731	-3.091	-2.747	0.4	-3.704	-3.061	-2.717	0.4	-3.698	-3.053	-2.712	0.4	-3.698	-3.053	-2.712
0.5	-3.884	-3.253	-2.913	0.5	-3.846	-3.218	-2.879	0.5	-3.842	-3.204	-2.872	0.5	-3.842	-3.204	-2.872
0.6	-4.012	-3.397	-3.055	0.6	-3.973	-3.353	-3.018	0.6	-3.969	-3.335	-3.012	0.6	-3.969	-3.335	-3.012
0.7	-4.143	-3.523	-3.187	0.7	-4.091	-3.477	-3.145	0.7	-4.084	-3.455	-3.137	0.7	-4.084	-3.455	-3.137
0.8	-4.248	-3.641	-3.310	0.8	-4.188	-3.587	-3.265	0.8	-4.181	-3.565	-3.254	0.8	-4.181	-3.565	-3.254
0.9	-4.350	-3.747	-3.422	0.9	-4.284	-3.692	-3.375	0.9	-4.263	-3.669	-3.361	0.9	-4.263	-3.669	-3.361
1.0	-4.448	-3.841	-3.522	1.0	-4.371	-3.786	-3.474	1.0	-4.348	-3.764	-3.464	1.0	-4.348	-3.764	-3.464

K = 2											
0.1	-2.862	-2.187	-1.828	0.1	-2.862	-2.184	-1.820	0.1	-2.869	-2.182	-1.818
0.2	-3.075	-2.406	-2.043	0.2	-3.079	-2.402	-2.039	0.2	-3.074	-2.398	-2.032
0.3	-3.236	-2.569	-2.206	0.3	-3.238	-2.564	-2.204	0.3	-3.238	-2.556	-2.198
0.4	-3.375	-2.704	-2.345	0.4	-3.367	-2.701	-2.341	0.4	-3.378	-2.693	-2.337
0.5	-3.496	-2.821	-2.464	0.5	-3.495	-2.818	-2.462	0.5	-3.486	-2.819	-2.459
0.6	-3.617	-2.930	-2.569	0.6	-3.590	-2.919	-2.572	0.6	-3.582	-2.924	-2.564
0.7	-3.713	-3.030	-2.666	0.7	-3.690	-3.015	-2.669	0.7	-3.680	-3.016	-2.662
0.8	-3.815	-3.119	-2.755	0.8	-3.780	-3.106	-2.756	0.8	-3.779	-3.100	-2.751
0.9	-3.900	-3.208	-2.837	0.9	-3.867	-3.194	-2.841	0.9	-3.851	-3.184	-2.834
1.0	-3.980	-3.286	-2.918	1.0	-3.932	-3.279	-2.919	1.0	-3.923	-3.267	-2.915

K = 3											
0.1	-2.809	-2.145	-1.785	0.1	-2.815	-2.141	-1.777	0.1	-2.831	-2.145	-1.777
0.2	-2.994	-2.341	-1.981	0.2	-3.000	-2.330	-1.977	0.2	-3.015	-2.332	-1.974
0.3	-3.138	-2.484	-2.128	0.3	-3.143	-2.473	-2.125	0.3	-3.146	-2.477	-2.123
0.4	-3.255	-2.602	-2.246	0.4	-3.257	-2.591	-2.246	0.4	-3.257	-2.595	-2.244

0.5	-3.362	-2.700	-2.348	0.5	-3.357	-2.692	-2.353	0.5	-3.361	-2.695	-2.352
0.6	-3.453	-2.790	-2.441	0.6	-3.443	-2.775	-2.448	0.6	-3.447	-2.784	-2.442
0.7	-3.538	-2.872	-2.522	0.7	-3.519	-2.858	-2.530	0.7	-3.518	-2.864	-2.524
0.8	-3.616	-2.945	-2.595	0.8	-3.588	-2.941	-2.604	0.8	-3.581	-2.937	-2.599
0.9	-3.691	-3.012	-2.666	0.9	-3.662	-3.006	-2.671	0.9	-3.661	-3.003	-2.662
1.0	-3.748	-3.079	-2.724	1.0	-3.739	-3.072	-2.734	1.0	-3.717	-3.067	-2.728

K = 4

0.1	-2.792	-2.129	1.772	0.1	-2.800	-2.126	-1.763	0.1	-2.805	-2.130	-1.763
0.2	-2.960	-2.315	-1.958	0.2	-2.974	-2.313	-1.952	0.2	-2.985	-2.311	-1.951
0.3	-3.096	-2.449	-2.100	0.3	-3.105	-2.451	-2.095	0.3	-3.108	-2.450	-2.095
0.4	-3.198	-2.560	-2.212	0.4	-3.209	-2.558	-2.211	0.4	-3.212	-2.564	-2.207
0.5	-3.294	-2.650	-2.307	0.5	-3.290	-2.650	-2.310	0.5	-3.296	-2.657	-2.307
0.6	-3.375	-2.733	-2.392	0.6	-3.372	-2.729	-2.398	0.6	-3.364	-2.739	-2.394
0.7	-3.451	-2.803	-2.468	0.7	-3.448	-2.800	-2.474	0.7	-3.424	-2.802	-2.470
0.8	-3.512	-2.869	-2.535	0.8	-3.506	-2.874	-2.541	0.8	-3.487	-2.867	-2.541
0.9	-3.562	-2.926	-2.596	0.9	-3.560	-2.936	-2.603	0.9	-3.552	-2.923	-2.601
1.0	-3.632	-2.978	-2.651	1.0	-3.623	-2.985	-2.659	1.0	-3.603	-2.983	-2.659

K = 5

0.1	-2.788	-2.121	-1.763	0.1	-2.791	-2.119	-1.756	0.1	-2.801	-2.121	-1.757
0.2	-2.955	-2.306	-1.949	0.2	-2.966	-2.302	-1.942	0.2	-2.971	-2.300	-1.938
0.3	-3.073	-2.436	-2.083	0.3	-3.091	-2.433	-2.082	0.3	-3.098	-2.431	-2.080
0.4	-3.176	-2.54	-2.193	0.4	-3.186	-2.541	-2.193	0.4	-3.191	-2.541	-2.192
0.5	-3.258	-2.628	-2.286	0.5	-3.273	-2.630	-2.290	0.5	-3.275	-2.630	-2.289
0.6	-3.336	-2.701	-2.366	0.6	-3.337	-2.702	-2.373	0.6	-3.337	-2.708	-2.370
0.7	-3.41	-2.774	-2.441	0.7	-3.403	-2.772	-2.448	0.7	-3.395	-2.777	-2.445
0.8	-3.477	-2.833	-2.505	0.8	-3.456	-2.838	-2.514	0.8	-3.457	-2.834	-2.511
0.9	-3.521	-2.889	-2.562	0.9	-3.499	-2.895	-2.573	0.9	-3.508	-2.887	-2.572
1.0	-3.584	-2.931	-2.617	1.0	-3.562	-2.943	-2.625	1.0	-3.552	-2.939	-2.620

Note: Critical values are obtained running 100,000 simulations.

2. For detrended series

K = 1				T = 250				T = 500			
ρ^2	10%	5%	1%	ρ^2	10%	5%	1%	ρ^2	10%	5%	1%
0.1	-3.292	-2.618	-2.263	0.1	-3.264	-2.593	-2.239	0.1	-3.254	-2.588	-2.234
0.2	-3.649	-2.994	-2.645	0.2	-3.616	-2.961	-2.613	0.2	-3.603	-2.955	-2.607
0.3	-3.911	-3.272	-2.932	0.3	-3.878	-3.234	-2.895	0.3	-3.853	-3.221	-2.886
0.4	-4.128	-3.501	-3.169	0.4	-4.088	-3.459	-3.126	0.4	-4.061	-3.435	-3.113
0.5	-4.313	-3.699	-3.369	0.5	-4.258	-3.650	-3.326	0.5	-4.238	-3.625	-3.306
0.6	-4.481	-3.868	-3.551	0.6	-4.405	-3.816	-3.500	0.6	-4.392	-3.794	-3.480
0.7	-4.623	-4.020	-3.711	0.7	-4.534	-3.964	-3.655	0.7	-4.530	-3.940	-3.634
0.8	-4.755	-4.166	-3.856	0.8	-4.661	-4.099	-3.798	0.8	-4.642	-4.071	-3.774
0.9	-4.882	-4.299	-3.991	0.9	-4.783	-4.219	-3.932	0.9	-4.749	-4.191	-3.902
1.0	-5.013	-4.415	-4.109	1.0	-4.875	-4.330	-4.045	1.0	-4.832	-4.302	-4.020

K = 2

0.1	-3.142	-2.453	-2.100	0.1	-3.115	-2.436	-2.078	0.1	-3.118	-2.438	-2.080
0.2	-3.461	-2.784	-2.426	0.2	-3.425	-2.751	-2.397	0.2	-3.429	-2.755	-2.399
0.3	-3.694	-3.031	-2.674	0.3	-3.655	-2.990	-2.642	0.3	-3.653	-2.996	-2.643
0.4	-3.901	-3.237	-2.878	0.4	-3.846	-3.190	-2.846	0.4	-3.835	-3.194	-2.843
0.5	-4.071	-3.414	-3.057	0.5	-4.004	-3.363	-3.021	0.5	-4.001	-3.365	-3.019
0.6	-4.233	-3.571	-3.219	0.6	-4.147	-3.518	-3.180	0.6	-4.144	-3.519	-3.177
0.7	-4.374	-3.718	-3.368	0.7	-4.284	-3.656	-3.329	0.7	-4.279	-3.653	-3.319
0.8	-4.500	-3.855	-3.506	0.8	-4.402	-3.786	-3.463	0.8	-4.394	-3.783	-3.450
0.9	-4.621	-3.975	-3.631	0.9	-4.508	-3.906	-3.585	0.9	-4.505	-3.895	-3.567
1.0	-4.742	-4.081	-3.754	1.0	-4.613	-4.018	-3.700	1.0	-4.600	-3.999	-3.681

K = 3

0.1	-3.059	-2.378	-2.020	0.1	-3.045	-2.368	-2.007	0.1	-3.055	-2.374	-2.005
0.2	-3.346	-2.667	-2.316	0.2	-3.327	-2.655	-2.298	0.2	-3.336	-2.655	-2.301
0.3	-3.560	-2.889	-2.537	0.3	-3.534	-2.870	-2.514	0.3	-3.548	-2.871	-2.521
0.4	-3.730	-3.072	-2.719	0.4	-3.713	-3.046	-2.695	0.4	-3.715	-3.046	-2.699
0.5	-3.876	-3.232	-2.877	0.5	-3.858	-3.198	-2.853	0.5	-3.856	-3.199	-2.851

0.6	-4.029	-3.369	-3.019	0.6	-3.999	-3.336	-2.993	0.6	-3.987	-3.335	-2.991
0.7	-4.162	-3.499	-3.150	0.7	-4.122	-3.461	-3.124	0.7	-4.111	-3.458	-3.117
0.8	-4.280	-3.618	-3.266	0.8	-4.231	-3.576	-3.238	0.8	-4.223	-3.570	-3.232
0.9	-4.400	-3.724	-3.378	0.9	-4.343	-3.683	-3.346	0.9	-4.321	-3.676	-3.341
1.0	-4.507	-3.824	-3.475	1.0	-4.426	-3.790	-3.454	1.0	-4.391	-3.779	-3.442

K = 4

0.1	-3.033	-2.346	-1.994	0.1	-3.011	-2.338	-1.981	0.1	-3.027	-2.347	-1.980
0.2	-3.297	-2.616	-2.271	0.2	-3.274	-2.611	-2.255	0.2	-3.281	-2.614	-2.263
0.3	-3.487	-2.830	-2.478	0.3	-3.473	-2.811	-2.458	0.3	-3.480	-2.815	-2.470
0.4	-3.641	-2.995	-2.646	0.4	-3.624	-2.977	-2.627	0.4	-3.627	-2.978	-2.639
0.5	-3.781	-3.141	-2.795	0.5	-3.755	-3.121	-2.775	0.5	-3.755	-3.122	-2.783
0.6	-3.910	-3.270	-2.925	0.6	-3.885	-3.242	-2.903	0.6	-3.887	-3.245	-2.910
0.7	-4.034	-3.382	-3.047	0.7	-3.990	-3.354	-3.019	0.7	-4.001	-3.359	-3.024
0.8	-4.144	-3.483	-3.151	0.8	-4.096	-3.454	-3.124	0.8	-4.103	-3.461	-3.131
0.9	-4.250	-3.581	-3.248	0.9	-4.201	-3.552	-3.225	0.9	-4.187	-3.558	-3.232
1.0	-4.340	-3.677	-3.336	1.0	-4.286	-3.645	-3.324	1.0	-4.261	-3.650	-3.320

K = 5

0.1	-3.012	-2.332	-1.977	0.1	-2.994	-2.325	-1.968	0.1	-3.015	-2.331	-1.966
0.2	-3.270	-2.595	-2.246	0.2	-3.247	-2.589	-2.234	0.2	-3.260	-2.592	-2.240
0.3	-3.453	-2.793	-2.449	0.3	-3.429	-2.784	-2.435	0.3	-3.446	-2.786	-2.438
0.4	-3.606	-2.955	-2.614	0.4	-3.581	-2.940	-2.599	0.4	-3.593	-2.944	-2.603
0.5	-3.736	-3.096	-2.754	0.5	-3.711	-3.074	-2.740	0.5	-3.724	-3.077	-2.741
0.6	-3.857	-3.219	-2.878	0.6	-3.821	-3.196	-2.863	0.6	-3.849	-3.198	-2.864
0.7	-3.964	-3.326	-2.989	0.7	-3.924	-3.305	-2.976	0.7	-3.946	-3.305	-2.976
0.8	-4.067	-3.418	-3.091	0.8	-4.016	-3.401	-3.077	0.8	-4.031	-3.403	-3.080
0.9	-4.156	-3.508	-3.181	0.9	-4.111	-3.487	-3.169	0.9	-4.107	-3.492	-3.173
1.0	-4.234	-3.589	-3.261	1.0	-4.192	-3.574	-3.259	1.0	-4.189	-3.575	-3.255

Note: Critical values are obtained running 100 000 simulations.

Table C: Critical values for the RALS-FKSS unit root test**1. For demeaned series**

K = 1				T = 100				T = 250				T = 500			
ρ^2	10%	5%	1%	ρ^2	10%	5%	1%	ρ^2	10%	5%	1%	ρ^2	10%	5%	1%
0.1	-2.964	-2.295	-1.939	0.1	-2.985	-2.303	-1.939	0.1	-2.985	-2.306	-1.939	0.1	-2.985	-2.306	-1.939
0.2	-3.221	-2.558	-2.209	0.2	-3.249	-2.570	-2.211	0.2	-3.250	-2.579	-2.212	0.2	-3.250	-2.579	-2.212
0.3	-3.410	-2.756	-2.410	0.3	-3.437	-2.769	-2.414	0.3	-3.436	-2.784	-2.420	0.3	-3.436	-2.784	-2.420
0.4	-3.558	-2.922	-2.577	0.4	-3.585	-2.938	-2.586	0.4	-3.592	-2.953	-2.595	0.4	-3.592	-2.953	-2.595
0.5	-3.674	-3.057	-2.723	0.5	-3.719	-3.079	-2.734	0.5	-3.715	-3.093	-2.747	0.5	-3.715	-3.093	-2.747
0.6	-3.788	-3.177	-2.852	0.6	-3.826	-3.199	-2.866	0.6	-3.836	-3.212	-2.879	0.6	-3.836	-3.212	-2.879
0.7	-3.893	-3.285	-2.964	0.7	-3.924	-3.308	-2.985	0.7	-3.929	-3.325	-3.003	0.7	-3.929	-3.325	-3.003
0.8	-3.984	-3.388	-3.068	0.8	-4.001	-3.407	-3.090	0.8	-4.013	-3.425	-3.109	0.8	-4.013	-3.425	-3.109
0.9	-4.068	-3.473	-3.162	0.9	-4.083	-3.495	-3.190	0.9	-4.097	-3.511	-3.207	0.9	-4.097	-3.511	-3.207
1.0	-4.139	-3.551	-3.246	1.0	-4.166	-3.577	-3.277	1.0	-4.172	-3.595	-3.295	1.0	-4.172	-3.595	-3.295

K = 2											
0.1	-2.896	-2.232	-1.874	0.1	-2.908	-2.238	-1.870	0.1	-2.920	-2.238	-1.880
0.2	-3.102	-2.458	-2.106	0.2	-3.131	-2.466	-2.107	0.2	-3.138	-2.473	-2.115
0.3	-3.259	-2.620	-2.275	0.3	-3.290	-2.634	-2.281	0.3	-3.302	-2.639	-2.292
0.4	-3.380	-2.748	-2.412	0.4	-3.406	-2.767	-2.427	0.4	-3.421	-2.775	-2.436
0.5	-3.481	-2.858	-2.529	0.5	-3.496	-2.877	-2.548	0.5	-3.531	-2.888	-2.556
0.6	-3.561	-2.952	-2.632	0.6	-3.576	-2.978	-2.653	0.6	-3.607	-2.988	-2.662
0.7	-3.636	-3.033	-2.723	0.7	-3.652	-3.063	-2.744	0.7	-3.671	-3.070	-2.757
0.8	-3.690	-3.105	-2.801	0.8	-3.710	-3.137	-2.828	0.8	-3.727	-3.145	-2.840
0.9	-3.751	-3.168	-2.869	0.9	-3.764	-3.198	-2.900	0.9	-3.785	-3.211	-2.913
1	-3.811	-3.219	-2.930	1.0	-3.837	-3.251	-2.965	1.0	-3.838	-3.268	-2.979

K = 3											
0.1	-2.857	-2.193	-1.834	0.1	-2.874	-2.193	-1.830	0.1	-2.869	-2.197	-1.837
0.2	-3.049	-2.400	-2.046	0.2	-3.064	-2.400	-2.049	0.2	-3.070	-2.408	-2.053
0.3	-3.178	-2.547	-2.199	0.3	-3.197	-2.546	-2.206	0.3	-3.212	-2.557	-2.212
0.4	-3.293	-2.659	-2.325	0.4	-3.298	-2.666	-2.336	0.4	-3.312	-2.678	-2.343

0.5	-3.385	-2.750	-2.428	0.5	-3.389	-2.769	-2.443	0.5	-3.399	-2.776	-2.448
0.6	-3.459	-2.831	-2.517	0.6	-3.463	-2.854	-2.535	0.6	-3.468	-2.858	-2.540
0.7	-3.527	-2.901	-2.591	0.7	-3.523	-2.927	-2.612	0.7	-3.537	-2.929	-2.621
0.8	-3.569	-2.963	-2.658	0.8	-3.585	-2.993	-2.686	0.8	-3.589	-2.995	-2.695
0.9	-3.614	-3.017	-2.716	0.9	-3.633	-3.048	-2.750	0.9	-3.630	-3.048	-2.756
1.0	-3.652	-3.071	-2.771	1.0	-3.672	-3.096	-2.803	1.0	-3.683	-3.098	-2.813

K = 4

0.1	-2.837	-2.175	-1.820	0.1	-2.852	-2.176	-1.817	0.1	-2.853	-2.187	-1.819
0.2	-3.019	-2.371	-2.020	0.2	-3.035	-2.377	-2.023	0.2	-3.042	-2.388	-2.030
0.3	-3.151	-2.508	-2.168	0.3	-3.162	-2.522	-2.175	0.3	-3.174	-2.527	-2.183
0.4	-3.247	-2.622	-2.287	0.4	-3.265	-2.635	-2.298	0.4	-3.277	-2.643	-2.309
0.5	-3.324	-2.714	-2.384	0.5	-3.340	-2.727	-2.403	0.5	-3.355	-2.742	-2.412
0.6	-3.391	-2.789	-2.469	0.6	-3.407	-2.809	-2.493	0.6	-3.423	-2.823	-2.501
0.7	-3.454	-2.854	-2.543	0.7	-3.468	-2.880	-2.568	0.7	-3.477	-2.894	-2.579
0.8	-3.513	-2.911	-2.607	0.8	-3.523	-2.938	-2.635	0.8	-3.531	-2.950	-2.643
0.9	-3.550	-2.956	-2.661	0.9	-3.578	-2.988	-2.695	0.9	-3.579	-3.003	-2.702
1.0	-3.581	-3.000	-2.703	1.0	-3.603	-3.041	-2.749	1.0	-3.609	-3.048	-2.757

K = 5

0.1	-2.827	-2.165	-1.810	0.1	-2.836	-2.168	-1.804	0.1	-2.849	-2.175	-1.809
0.2	-3.000	-2.357	-2.010	0.2	-3.013	-2.364	-2.008	0.2	-3.030	-2.370	-2.019
0.3	-3.125	-2.492	-2.154	0.3	-3.139	-2.506	-2.156	0.3	-3.158	-2.514	-2.166
0.4	-3.218	-2.598	-2.267	0.4	-3.231	-2.612	-2.276	0.4	-3.259	-2.625	-2.286
0.5	-3.307	-2.684	-2.363	0.5	-3.307	-2.705	-2.380	0.5	-3.334	-2.715	-2.389
0.6	-3.368	-2.760	-2.443	0.6	-3.373	-2.782	-2.470	0.6	-3.402	-2.797	-2.476
0.7	-3.425	-2.823	-2.515	0.7	-3.431	-2.848	-2.542	0.7	-3.449	-2.862	-2.551
0.8	-3.472	-2.873	-2.577	0.8	-3.474	-2.908	-2.608	0.8	-3.494	-2.918	-2.618
0.9	-3.513	-2.922	-2.630	0.9	-3.516	-2.955	-2.668	0.9	-3.547	-2.970	-2.679
1.0	-3.551	-2.968	-2.675	1.0	-3.548	-3.006	-2.711	1.0	-3.571	-3.012	-2.727

Note: Critical values are obtained running 100,000 simulations.

2. For detrended series

K = 1				T = 250				T = 500			
ρ^2	10%	5%	1%	ρ^2	10%	5%	1%	ρ^2	10%	5%	1%
0.1	-3.184	-2.520	-2.167	0.1	-3.184	-2.517	-2.163	0.1	-3.189	-2.517	-2.163
0.2	-3.500	-2.859	-2.509	0.2	-3.503	-2.856	-2.506	0.2	-3.503	-2.856	-2.511
0.3	-3.732	-3.103	-2.765	0.3	-3.739	-3.110	-2.762	0.3	-3.731	-3.106	-2.769
0.4	-3.924	-3.304	-2.974	0.4	-3.922	-3.311	-2.972	0.4	-3.918	-3.313	-2.980
0.5	-4.087	-3.473	-3.152	0.5	-4.078	-3.477	-3.149	0.5	-4.075	-3.487	-3.162
0.6	-4.224	-3.622	-3.307	0.6	-4.216	-3.625	-3.309	0.6	-4.222	-3.636	-3.322
0.7	-4.353	-3.753	-3.444	0.7	-4.338	-3.759	-3.450	0.7	-4.353	-3.767	-3.464
0.8	-4.471	-3.877	-3.569	0.8	-4.448	-3.877	-3.579	0.8	-4.459	-3.888	-3.595
0.9	-4.574	-3.987	-3.681	0.9	-4.551	-3.984	-3.699	0.9	-4.557	-4.004	-3.711
1.0	-4.679	-4.079	-3.785	1.0	-4.634	-4.085	-3.806	1.0	-4.664	-4.106	-3.821

K = 2

0.1	-3.077	-2.406	-2.049	0.1	-3.068	-2.399	-2.040	0.1	-3.072	-2.406	-2.049
0.2	-3.359	-2.703	-2.352	0.2	-3.353	-2.697	-2.346	0.2	-3.363	-2.707	-2.355
0.3	-3.569	-2.921	-2.574	0.3	-3.561	-2.916	-2.573	0.3	-3.568	-2.932	-2.587
0.4	-3.737	-3.100	-2.756	0.4	-3.736	-3.095	-2.759	0.4	-3.741	-3.112	-2.775
0.5	-3.875	-3.247	-2.918	0.5	-3.870	-3.248	-2.920	0.5	-3.884	-3.266	-2.932
0.6	-3.998	-3.379	-3.056	0.6	-3.992	-3.382	-3.060	0.6	-4.004	-3.405	-3.073
0.7	-4.117	-3.497	-3.177	0.7	-4.096	-3.500	-3.188	0.7	-4.119	-3.520	-3.199
0.8	-4.211	-3.599	-3.286	0.8	-4.201	-3.606	-3.303	0.8	-4.214	-3.628	-3.314
0.9	-4.310	-3.696	-3.386	0.9	-4.299	-3.703	-3.406	0.9	-4.303	-3.722	-3.419
1.0	-4.397	-3.779	-3.476	1.0	-4.390	-3.797	-3.501	1.0	-4.382	-3.812	-3.517

K = 3

0.1	-3.023	-2.348	-1.997	0.1	-3.022	-2.345	-1.991	0.1	-3.035	-2.361	-1.995
0.2	-3.279	-2.617	-2.272	0.2	-3.277	-2.622	-2.276	0.2	-3.299	-2.636	-2.278
0.3	-3.476	-2.816	-2.477	0.3	-3.467	-2.825	-2.484	0.3	-3.492	-2.834	-2.490
0.4	-3.617	-2.978	-2.646	0.4	-3.616	-2.993	-2.654	0.4	-3.648	-3.000	-2.664
0.5	-3.739	-3.117	-2.789	0.5	-3.742	-3.128	-2.802	0.5	-3.784	-3.140	-2.811

0.6	-3.852	-3.233	-2.914	0.6	-3.852	-3.249	-2.928	0.6	-3.893	-3.264	-2.935
0.7	-3.951	-3.342	-3.023	0.7	-3.959	-3.358	-3.044	0.7	-3.985	-3.372	-3.048
0.8	-4.042	-3.435	-3.121	0.8	-4.054	-3.453	-3.142	0.8	-4.086	-3.472	-3.148
0.9	-4.124	-3.519	-3.211	0.9	-4.141	-3.539	-3.236	0.9	-4.161	-3.560	-3.248
1.0	-4.201	-3.597	-3.291	1.0	-4.210	-3.621	-3.325	1.0	-4.218	-3.638	-3.339

K = 4

0.1	-3.004	-2.335	-1.982	0.1	-3.000	-2.335	-1.977	0.1	-3.011	-2.345	-1.982
0.2	-3.253	-2.598	-2.249	0.2	-3.249	-2.603	-2.249	0.2	-3.258	-2.611	-2.255
0.3	-3.433	-2.790	-2.447	0.3	-3.430	-2.797	-2.450	0.3	-3.442	-2.811	-2.459
0.4	-3.570	-2.945	-2.610	0.4	-3.573	-2.954	-2.616	0.4	-3.601	-2.966	-2.625
0.5	-3.690	-3.077	-2.748	0.5	-3.695	-3.087	-2.755	0.5	-3.726	-3.093	-2.768
0.6	-3.793	-3.190	-2.866	0.6	-3.792	-3.200	-2.878	0.6	-3.837	-3.209	-2.893
0.7	-3.886	-3.284	-2.967	0.7	-3.889	-3.299	-2.987	0.7	-3.927	-3.311	-3.000
0.8	-3.966	-3.373	-3.061	0.8	-3.971	-3.385	-3.084	0.8	-3.998	-3.398	-3.100
0.9	-4.050	-3.452	-3.143	0.9	-4.048	-3.465	-3.170	0.9	-4.071	-3.480	-3.188
1.0	-4.133	-3.529	-3.222	1.0	-4.121	-3.541	-3.248	1.0	-4.129	-3.562	-3.266

K = 5

0.1	-2.998	-2.322	-1.973	0.1	-2.992	-2.324	-1.968	0.1	-3.008	-2.333	-1.977
0.2	-3.244	-2.580	-2.239	0.2	-3.235	-2.586	-2.236	0.2	-3.264	-2.592	-2.248
0.3	-3.423	-2.768	-2.434	0.3	-3.415	-2.780	-2.435	0.3	-3.443	-2.784	-2.450
0.4	-3.554	-2.921	-2.593	0.4	-3.557	-2.930	-2.597	0.4	-3.584	-2.942	-2.610
0.5	-3.660	-3.050	-2.726	0.5	-3.674	-3.059	-2.734	0.5	-3.697	-3.070	-2.748
0.6	-3.752	-3.160	-2.839	0.6	-3.774	-3.169	-2.854	0.6	-3.799	-3.186	-2.866
0.7	-3.846	-3.253	-2.943	0.7	-3.861	-3.264	-2.960	0.7	-3.884	-3.284	-2.971
0.8	-3.921	-3.341	-3.032	0.8	-3.941	-3.352	-3.053	0.8	-3.970	-3.372	-3.064
0.9	-3.989	-3.416	-3.113	0.9	-4.013	-3.425	-3.135	0.9	-4.039	-3.445	-3.151
1.0	-4.067	-3.480	-3.184	1.0	-4.083	-3.505	-3.212	1.0	-4.091	-3.518	-3.228

Note: Critical values are obtained running 100 000 simulations.

Source of tables A–C: Authors' own calculations

Appendix II.

This study investigates the stationarity of real interest rate differentials (RIDs) using Germany as the benchmark country. The objective is to assess the degree of financial market integration among European nations.

We first test the normality of real interest differentials (RIDs) by employing Shapiro–Wilk normality test and the results are presented in the table below.

Table A: Shapiro–Wilk normality test

Country	Test stat.	<i>p</i> -values
France	0.835*	0
Italy	0.769*	0
Spain	0.889*	0
UK	0.922*	0

Note: * represents significance at the 1% level.

The results presented in Table II. A indicate that none of the real interest rate differential (RID) series exhibit a normal distribution. Following the assessment of normality, the linearity of the RID series is examined. The findings of this analysis are displayed in the subsequent table.

Table B: Harvey *et al.* (2008) Linearity test results

Country	Test stat.	<i>p</i> -values
France	4.185	0.123
Italy	6.518**	0.038
Spain	19.088*	0.000
UK	3.995	0.136

Note: * and ** represents significance at the 1% and 5% levels, respectively.

The findings presented in Table B reveal nonlinearity in the real interest rate differentials (RIDs) of Italy and Spain. Consequently, the stochastic properties of these RIDs are investigated using unit root tests based on ESTAR models. For the remaining series, which exhibit linearity, unit root tests that based on linear models are employed. The results of the appropriate unit root tests are displayed in the following table.

Table C: RALS-FADF and RALS-FKSS test results

Country	FADF-RALS	FKSS-RALS	Frequency	Opt. lag	ρ	F-test stat
France	-6.441*	–	1	0	0.783	13.626*
Italy	–	-6.776*	1	0	0.862	8.987*
Spain	–	-2.195	1	14	0.816	26.146*
UK	-8.300*	–	1	1	0.873	1.928

Note: * represents significance at the 1% level.

Given that the unit root tests demonstrate stationarity for the real interest rate differentials (RIDs) of France, Italy, and the United Kingdom, the statistical significance of trigonometric terms can be assessed for these countries. The F-test statistics indicate significance for France and Italy, while the UK exhibits non-significance. Conversely, the non-stationary nature of Spain's RIDs precludes definitive conclusions regarding the significance of the Fourier function. Therefore, stationarity is confirmed for the RIDs of France and Italy. To examine the null hypothesis of a unit root for the RIDs of Spain and the UK, the RALS-KSS and RALS-ADF unit root tests are employed, respectively.

Table D: RALS-ADF and RALS-KSS test results

Country	ADF-RALS	KSS-RALS	Opt. lag	ρ
Spain	–	1.793	14	0.119
UK	0.272	–	14	0.892

Source of tables A–D: Authors' own calculations

The results of the aforementioned analyses in Table III. D indicate that both series exhibit a unit root.

Appendix III

To ensure the robustness of the findings, the stochastic properties of the RIDs for the EU5 countries are re-evaluated using a more recent data sample 1950 to 2018. Initially, the normality of the series is assessed employing the Shapiro–Wilk test. The results of this analysis are presented in the following table.

Table A: Shapiro-Wilk normality test

Country	Test stat.	<i>p</i> -values
France	0.860	0.000
Germany	0.910	0.000
Italy	0.941	0.003
Spain	0.915	0.000
UK	0.937	0.002

Note: * represents significance at the 1% level.

The results presented in Table III. A demonstrate that all series exhibit a non-normal distribution. Subsequently, the linearity of these series is examined utilizing the test developed by Harvey *et al.* (2008).

Table B: Harvey *et al.* (2008) linearity test results

Country	Test stat.	<i>p</i> -values
France	15.328*	0.000
Germany	2.518	0.284
Italy	9.505*	0.009
Spain	0.475	0.789
UK	1.252	0.535

Note: * represents significance at the 1% level.

The results displayed in Table III. B indicate that only the RID series of France and Italy exhibit nonlinearity, while the remaining series demonstrate linearity. Consequently, the unit root properties of the RIDs for France and Italy are examined using unit root tests specifically designed for nonlinear models. Conversely, for the remaining series, stationarity is assessed employing unit root tests based on linear models:

Table C: RALS-FADF and RALS-FKSS test results

Country	FADF-RALS	FKSS-RALS	Frequency	Opt. lag	ρ	F-test stat
France	–	–0.164	1	10	0.947	3.157
Germany	–3.229	–	1	0	0.849	4.110
Italy	–	–5.473*	3	0	0.939	4.975**
Spain	–4.255*	–	3	0	0.731	6.403**
UK	–2.802***	–	2	1	0.786	5.283**

Note: * represents significance at the 1% level.

The results presented in Table III. C indicate stationarity for the RID series of Italy, Spain, and the United Kingdom, with statistically significant trigonometric terms. However, the unit root test statistics for the remaining RID series suggest the presence of a unit root, precluding a definitive conclusion regarding their stationarity. Furthermore, the non-stationarity of these series prohibits the use of *F*-test statistics to assess the significance of trigonometric terms. Therefore, the stationarity of these series is further investigated using the RALS unit root test without the inclusion of a Fourier function. The results of this analysis are presented in the following table.

Table D: RALS-ADF and RALS-KSS test results

Country	ADF-RALS	KSS-RALS	Opt. lag	ρ
France	–	–0.264	5	0.999
Germany	–6.088*	–	0	0.848

Source of tables A–D: Authors' own calculations

The findings presented in Table III. D demonstrate that the RIDs of Germany exhibit stationarity, while the RIDs of France indicate the presence of a unit root.

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